Materials

Strength of materials

UNIVERSITATEA BABEȘ-BOLYAI

Last update: March 6, 2023

Agenda

- Strength of materials theory
- Force, Stress, and Strain
- Material properties
- Characterisation
- Biological tissues of interest

Strength of materials

Definition

The study of describing the amount of load that can be exerted on a material until it deforms or fails.

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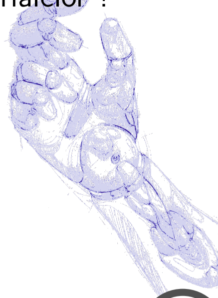
What concepts you remember from 'Rezistența Materialelor'?



Strength of materials

General overview

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Resistance relates to the amount of load we exert on the object.

Deformation can be either temporary or permanent.

Strength of materials

Stress

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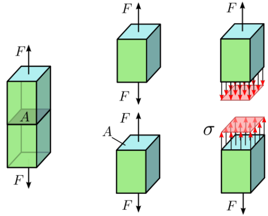
$$\sigma = \frac{F}{A}$$



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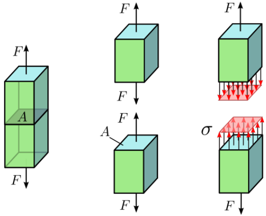


Why use stress instead of force?

Strength of materials

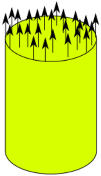
Stress

Stress can be either *normal* or *shear* stress.

Strength of materials

Stress

Stress can be either *normal* or *shear* stress. Normal stress acts perpendicular to a surface



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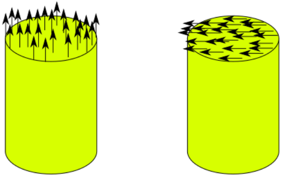

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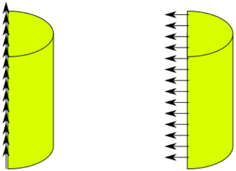

What is *normal* or *parallel*, depends what is the surface of reference

Strength of materials

Strain

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$$\epsilon = \frac{\Delta(x-X)}{\Delta(X)}$$

Strength of materials

Tensile tests

How do we quantify the properties of a material?

Strength of materials

Tensile tests

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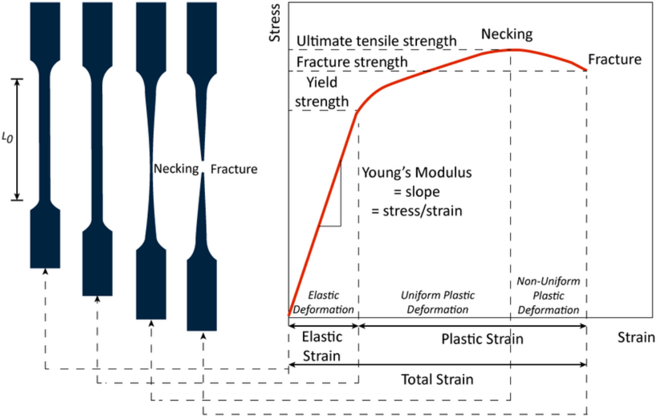
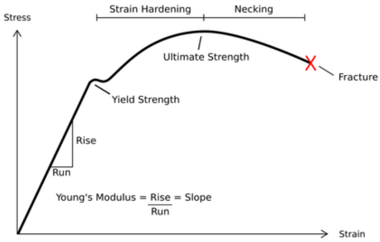


Strength of materials

Strain-stress graph



How does it help? What kind of information does it provide?

Material properties of interest

Stiffness

We can derive several information from this graph. Most important property is called *Stiffness* or *Elasticity*. It is calculated as the slope of the stress-strain curve during the linear part.

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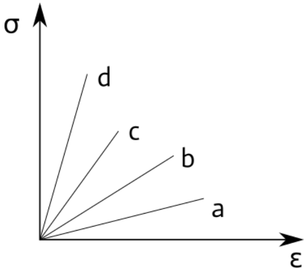

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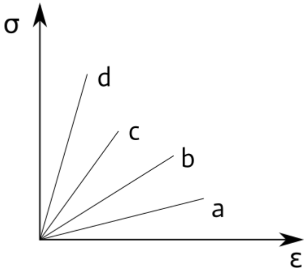

Which of these materials is more *stiff*?

Stiffness

Modeling

For the linear deformation, we can model the material as a perfect spring, following **Hooke's law**.

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What are the units of E (called *Young's modulus*)?

Material properties of interest

Yield point

Elastic deformation

Deformation that disappears after the forces are removed

Plastic deformation

Deformation that remains after the forces are removed

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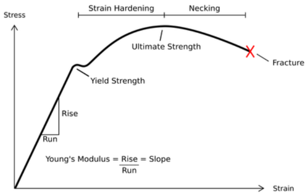
Deformation that remains after the forces are removed

Every material has a limit for elastic/plastic deformation, called the Yield point.



Material properties of interest

Plastic deformation

What happens when the yield point is exceeded?

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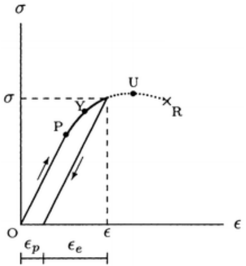

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Material properties of interest

Plastic deformation

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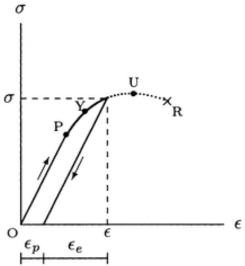

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Residual (plastic) deformation ϵ_p

Material properties of interest

Plastic deformation

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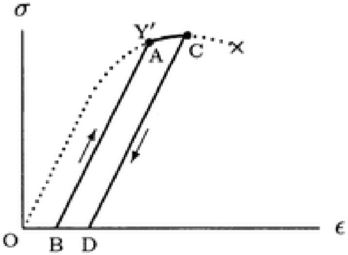

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Plastic deformation can add up

Material properties of interest

Viscoelasticity

Not all materials behave like perfect springs: the case of Viscosity.

$$F = \mu A \frac{u}{y}$$

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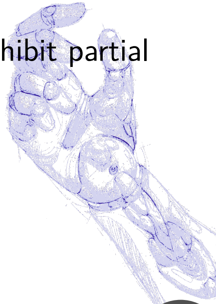
If a material has partial fluid like properties, it can exhibit partial viscosity, together with their elastic properties



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Viscoelasticity!

Viscoelasticity

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Deformation depends on velocity of strain/stress.

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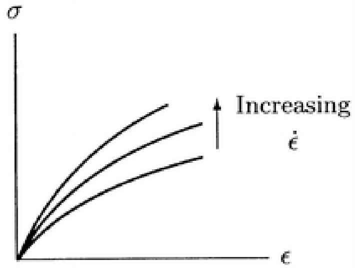

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Viscoelasticity

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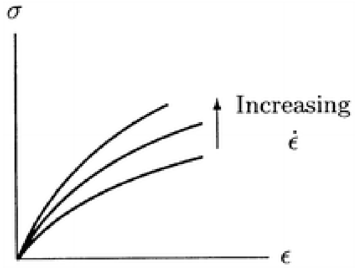

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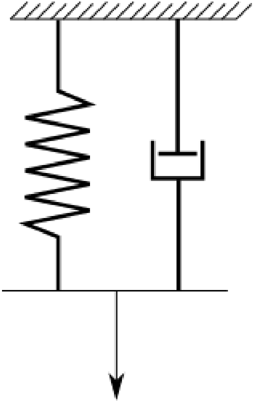
If we model an elastic material as a spring, what do we use for viscous materials?

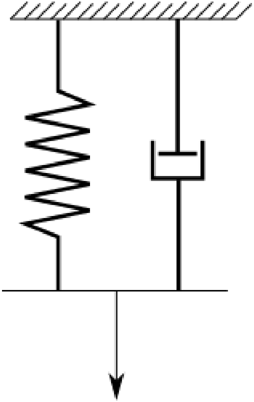
Viscoelasticity

Modeling



Viscoelasticity

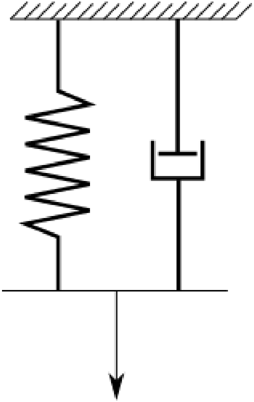
Modeling



The spring element, models the elastic behaviour of the material

Viscoelasticity

Modeling



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The dashpot element, models the viscous behaviour of the material

Viscoelasticity

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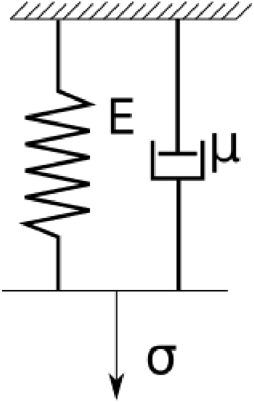

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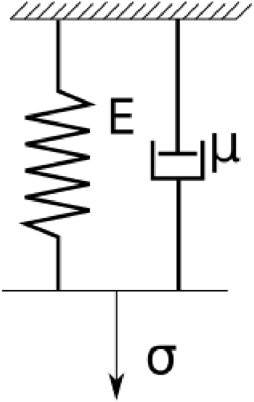
$$\sigma = E\epsilon + \mu\frac{d\epsilon}{dt}$$

Viscoelasticity

Modeling



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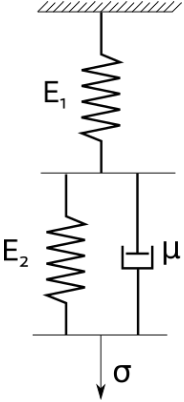
$\sigma = E\epsilon + \mu\frac{d\epsilon}{dt}$
No known materials that follow this relationship

Viscoelasticity

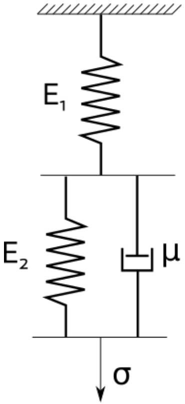
Standard solid model



A more complex and realistic model for viscoelastic materials

Viscoelasticity

Standard solid model

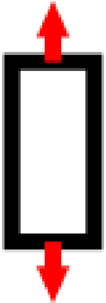


A more complex and realistic model for viscoelastic materials

$$(E_1 + E_2)\sigma + \mu\frac{d\sigma}{dt} = E_1 E_2 \epsilon + E_1 \mu \frac{d\epsilon}{dt}$$

Material properties

Types of load



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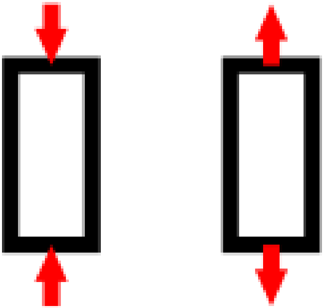

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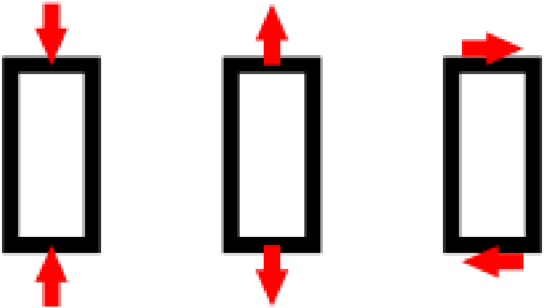

Material properties

Types of load

Do we need extra testing for different directions?

Strength of materials

Isotropic vs Anisotropic

Isotropic material

A material that has the same properties regardless of the axis of measurement

Anisotropic material

A material that its properties differ along different axes.

Strength of materials

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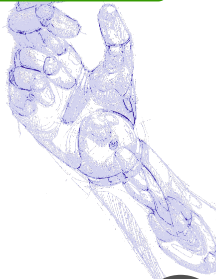
What can be the source of anisotropy?



Strength of materials

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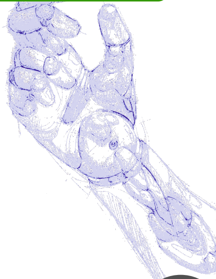
What can be the source of anisotropy?
What kind do you think biological materials are?



Biologic materials

Bones

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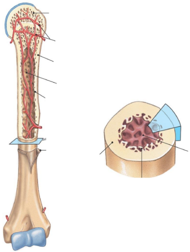
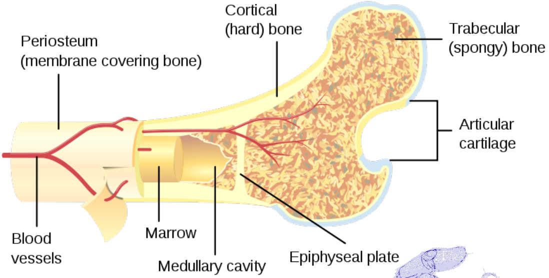


It is very nonhomogeneous, comprising of different materials at different scales.

Biologic materials

Bones

Biggest division of bones is in cortical and trabecular bone



Biologic materials

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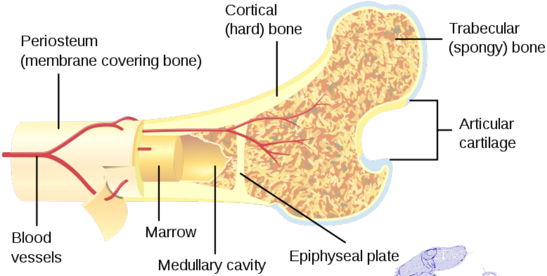

Biologic materials

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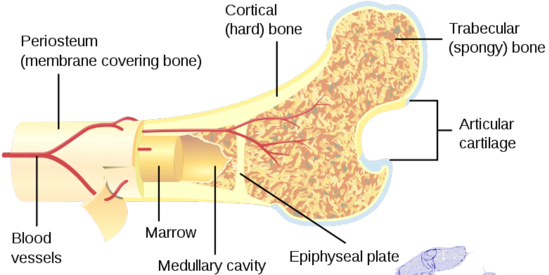

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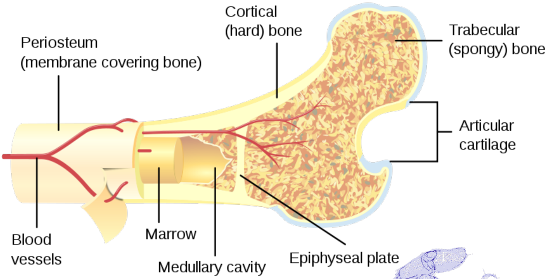

The material properties of bone vary vastly based on age, sex, person, bone location, pathology

Biologic materials

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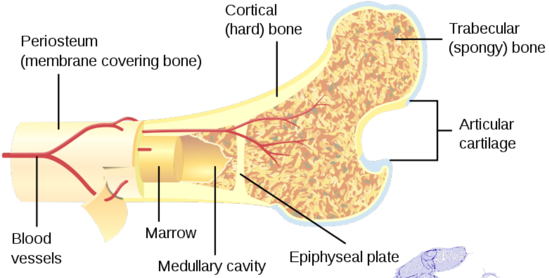


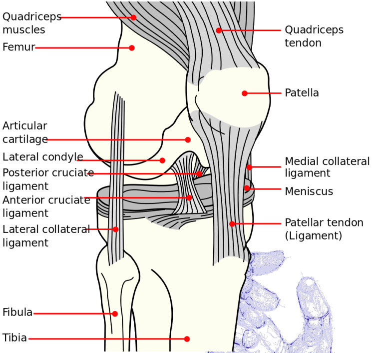
The material properties of bone vary vastly based on age, sex, person, bone location, pathology

They are usually modeled as viscoelastic materials

Biologic materials

Tendons and Ligaments

Ligaments are keeping bones together, allowing motion between then only on some degrees of freedom.



Biologic materials

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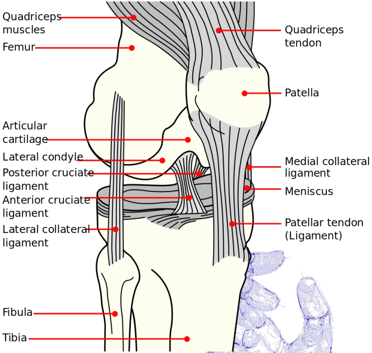

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Biologic materials

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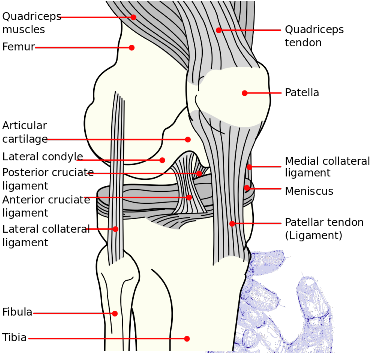

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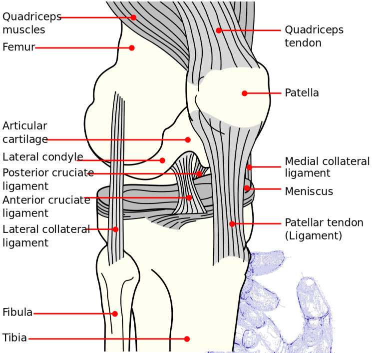
Biologic materials

Tendons and Ligaments



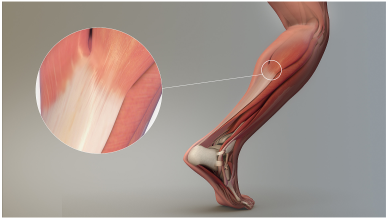
Ligaments are keeping bones together, allowing motion between then only on some degrees of freedom.

They are considered almost totally elastic at strains up to 0.25 however, viscous effects appear at higher strains. This is due to the components of the ligaments that are being streched at different strains.

Biologic materials

Tendons and Ligaments

Tendons is the connecting tissue between muscles and bones.



Biologic materials

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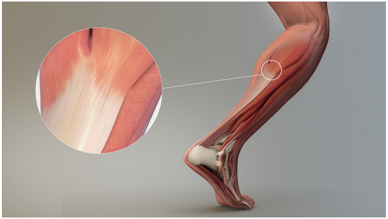

Tendons have similar behaviour and properties as ligaments, though they stop being elastic at lower strains (0.05).

There are three types of muscles:

Biologic materials

Muscles

There are three types of muscles:

- Skeletal

Biologic materials

Muscles

There are three types of muscles:

- Skeletal
- Smooth

Biologic materials

Muscles

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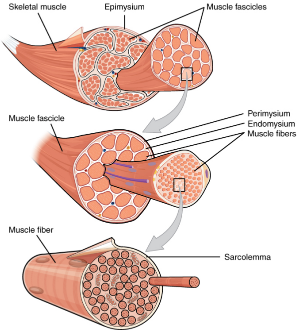
- Skeletal
- Smooth
- Cardiac

Biologic materials

Muscles



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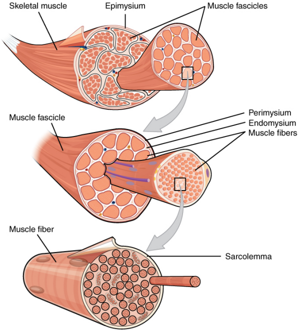
Very complex structure of fibers bundled together

Biologic materials

Muscles



There are three types of muscles:

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Very complex structure of fibers bundled together
They generate force by contracting and relaxing.

Material properties

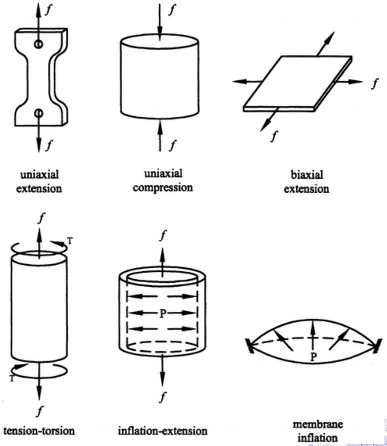
How do we calculate them?

Looking for a relationship between stress and strain, under different directions.

Material properties

How do we calculate them?

Looking for a relationship between stress and strain, under different directions.
Based on the type of tissue and on the material property we are interested in

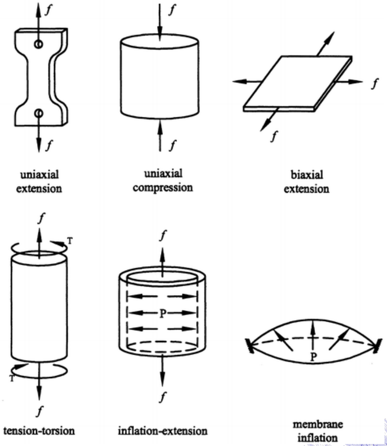


Adapted from 'An introduction to biomechanics', by Humphrey et al.

Material properties

How do we calculate them?

Looking for a relationship between stress and strain, under different directions.
Based on the type of tissue and on the material property we are interested in
For viscoelastic materials, test must be performed under different velocities



Adapted from 'An introduction to biomechanics', by Humphrey et al.

Overview

- Strength of materials

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- Stress, strain

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- Normal, shear strength

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- Material testing

Coming up next

Finite element modeling

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Finite element modeling, or how do we use the material properties in non-standard object forms.

Questions?