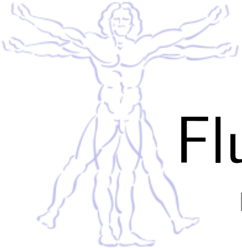


Fluid biomechanics

Pressure, flow rate, assumptions

UNIVERSITATEA BABEȘ-BOLYAI

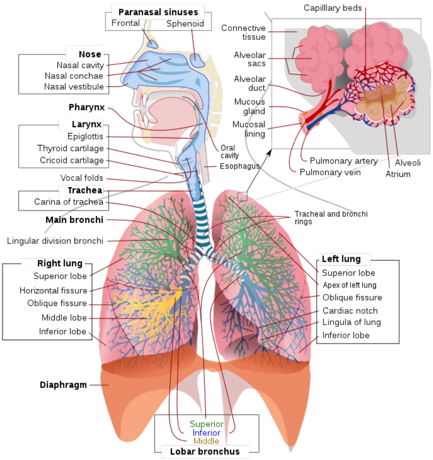
Last update: March 28, 2023

Agenda

- Fluid systems of the human body
- Fluid mechanics principles
- Blood
- Fluid solid interactions
- CFD and FEM

Fluid systems

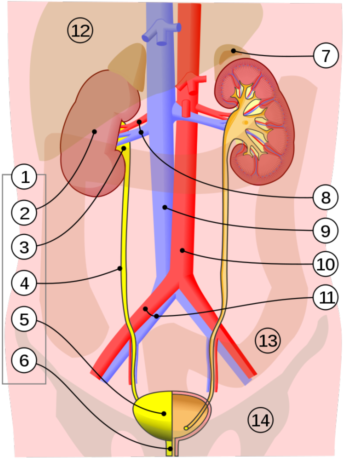
Respiratory system



From *Wikipedia: Respiratory system*

Fluid systems

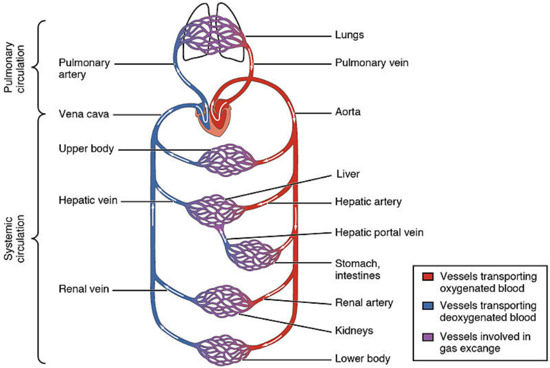
Urinary system



From *Wikipedia: Urinary system*

Fluid systems

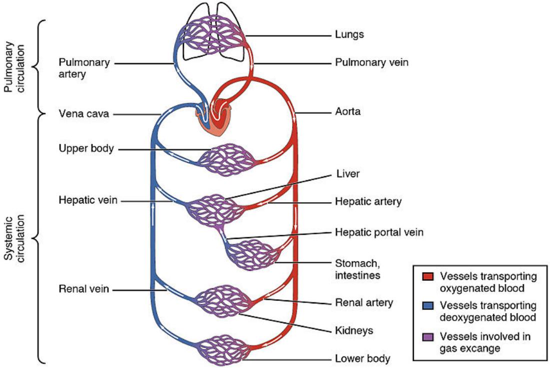
Cardiovascular system



From *Anatomy & Physiology, Connexions Web site*

Fluid systems

Cardiovascular system

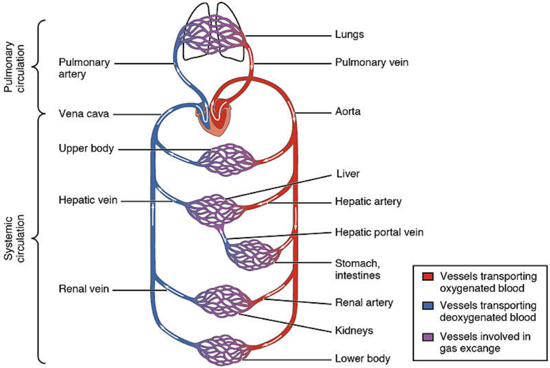


From *Anatomy & Physiology, Connexions Web site*

- Pressure in vessels

Fluid systems

Cardiovascular system



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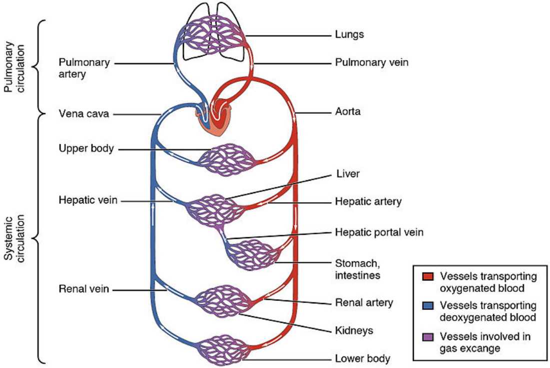
- Pressure in vessels
- Blood flow rate

Fluid systems

Cardiovascular system



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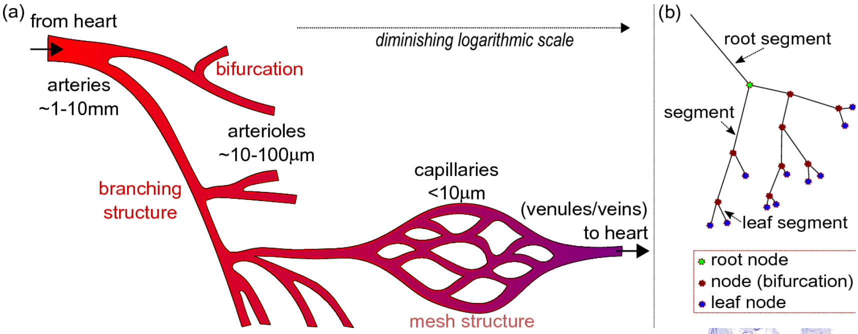
- Pressure in vessels
- Blood flow rate
- Turbulence

Fluid systems

Cardiovascular system



Incomplressible flow equation

Daniel Bernoulli 1700-1782

Fluid mechanics

Basic principles

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$$\frac{u^2}{2} + gz + \frac{p}{\rho} = constant$$

Fluid mechanics

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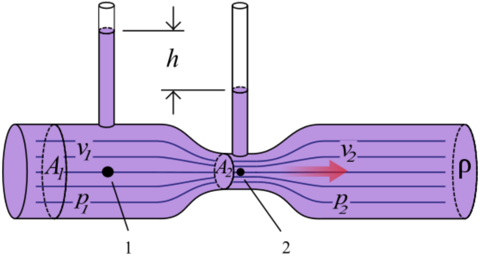
$$\frac{u^2}{2} + gz + \frac{p}{\rho} = constant$$

u: fluid flow speed
g: gravitational acceleration
z: elevation
p: pressure
ρ: fluid density

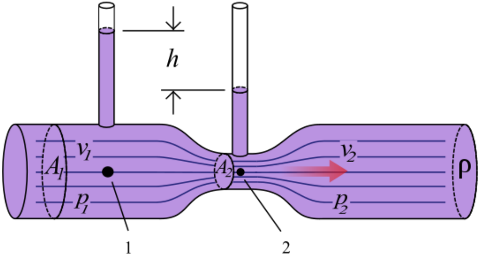
Fluid mechanics

Venturi effect



Fluid mechanics

Venturi effect



$$\frac{u^2}{2} + gz + \frac{p}{\rho} = constant$$

Fluid mechanics

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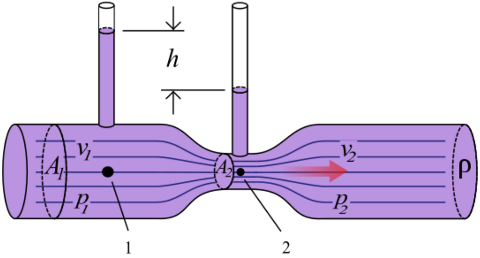

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We assume incompressible flow, therefore equal flow rate (Conservation of mass)

Fluid mechanics

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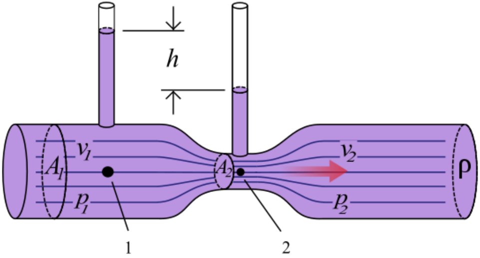

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$$Q_1 = u_1 A_1 = u_2 A_2 = Q_2$$

Fluid mechanics

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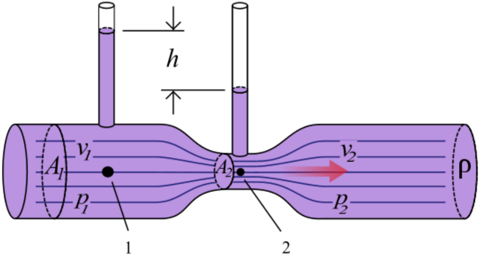

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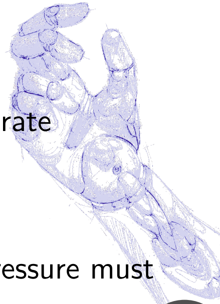
If area decreases, velocity increases. Bernoulli says pressure must drop.



Fluid mechanics

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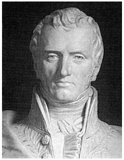
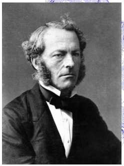
Which fluid property does Bernoulli neglect?

Fluid mechanics

Navier-Stokes equations



Claude-Lois Navier

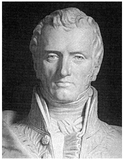


Sir George Stokes

Fluid mechanics

Navier-Stokes equations

$$\nabla \vec{u} = 0$$



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Fluid mechanics

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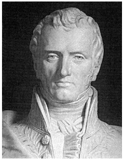
$$\nabla \vec{u} = 0$$

$$\rho \frac{\partial u}{\partial t} + \rho \vec{u} \nabla \vec{u} = -\nabla p + \mu \nabla^2 \vec{u} + \rho F$$



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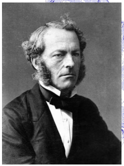

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Fluid mechanics

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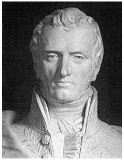

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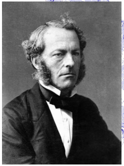

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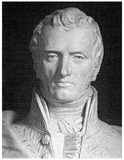
(Newton's second law F=ma)



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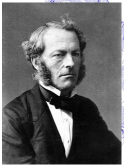

Sir George Stokes

Fluid mechanics

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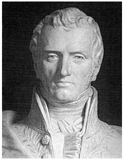
(Newton's second law F=ma)

We don't understand these fully!



Claude-Lois Navier

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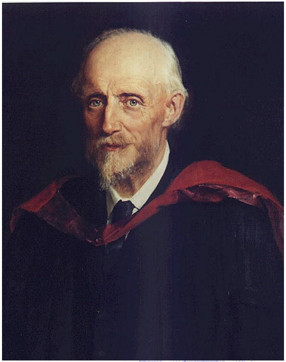
Fluid mechanics

Reynolds number

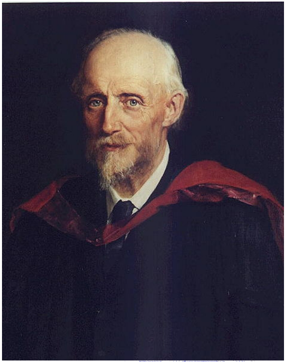


Osborne Reynolds 1842-1912

Fluid mechanics

Reynolds number

$$Re = \frac{\rho u L}{\mu}$$

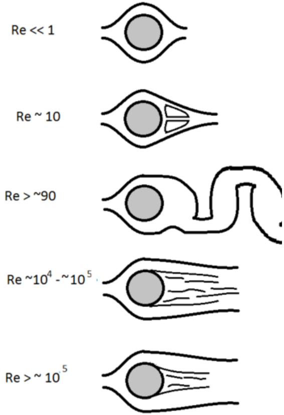


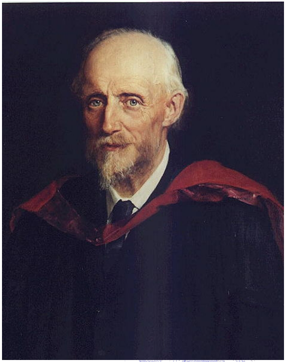
Osborne Reynolds 1842-1912

Fluid mechanics

Reynolds number

$$Re = \frac{\rho u L}{\mu} = \frac{F_{inertia}}{F_{viscous}}$$





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Fluid mechanics

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$$\rho\frac{\partial u}{\partial t} + \rho\vec{u}\nabla\vec{u} = -\nabla p + \mu\nabla^2\vec{u} + \rho F$$

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Fluid mechanics

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Fluid mechanics

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For Re $\gg$1:

Fluid mechanics
Reynolds number

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For Re $\ll$1:

$$\rho\frac{\partial u}{\partial t}+\nabla p=+\mu\nabla^2\vec{u}$$

For Re $\gg$1:

$$\rho\frac{\partial u}{\partial t}+\rho\vec{u}\nabla\vec{u}=-\nabla p$$

Fluid mechanics

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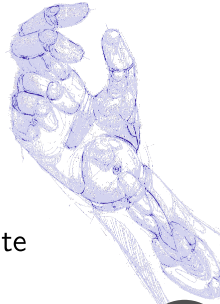
For Re $\gg$1:

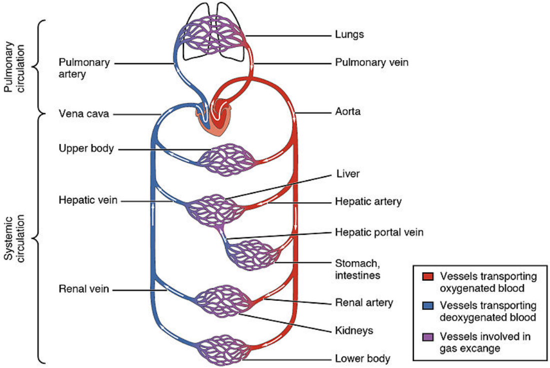
$$\rho\frac{\partial u}{\partial t} + \rho\vec{u}\nabla\vec{u} = -\nabla p$$

Either of these are much simpler to compute



Fluid mechanics

Reynolds number in cardiovascular system



From *Anatomy & Physiology, Connexions Web site*

Fluid mechanics

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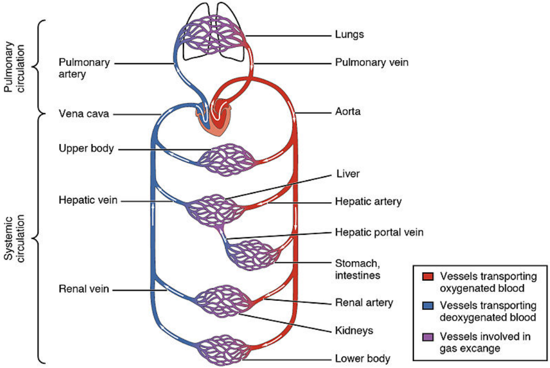
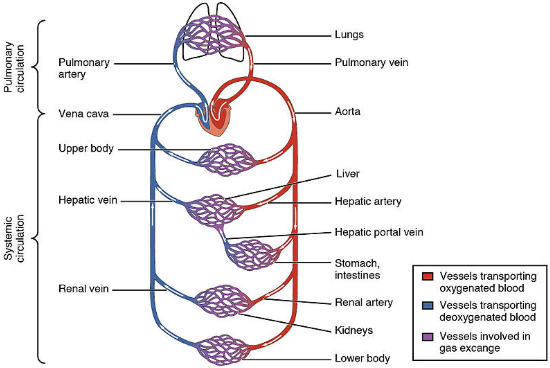


From *Anatomy & Physiology, Connexions Web site*

- Ascending Aorta: 4500

Fluid mechanics

Reynolds number in cardiovascular system

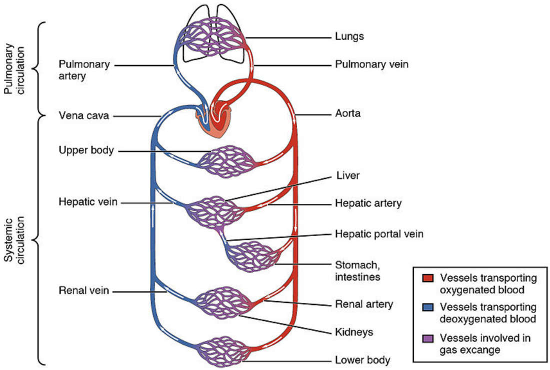


From *Anatomy & Physiology, Connexions Web site*

- Ascending Aorta: 4500
- Descending Aorta: 3400

Fluid mechanics

Reynolds number in cardiovascular system

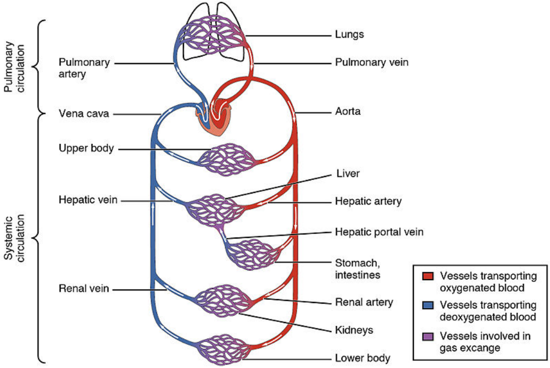


From *Anatomy & Physiology, Connexions Web site*

- Ascending Aorta: 4500
- Descending Aorta: 3400
- Abdominal Aorta: 1250

Fluid mechanics

Reynolds number in cardiovascular system

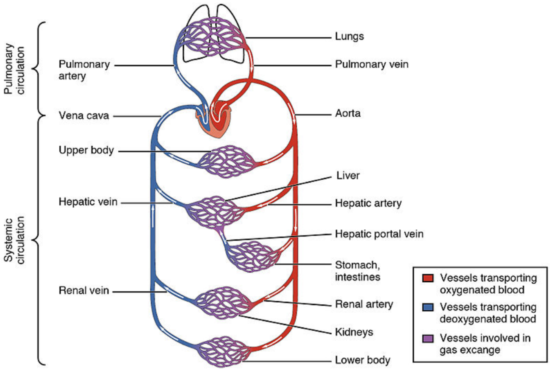


From *Anatomy & Physiology, Connexions Web site*

- Ascending Aorta: 4500
- Descending Aorta: 3400
- Abdominal Aorta: 1250
- Femoral artery: 1000

Fluid mechanics

Reynolds number in cardiovascular system



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- Ascending Aorta: 4500
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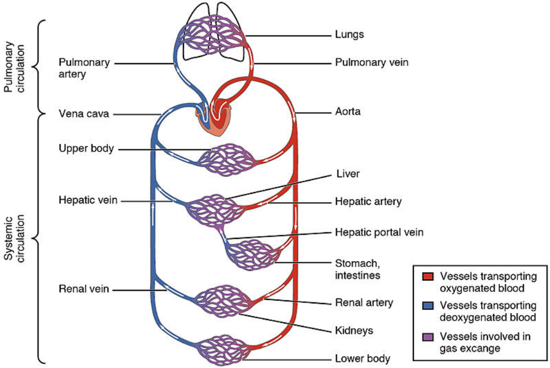
- Arteriole: 0.09

Fluid mechanics

Reynolds number in cardiovascular system



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- Arteriole: 0.09
- Capilaries: 0.001

Fluid mechanics

What about biology?

Can we use such equations for blood flow?

What are the assumptions of these equations?

Fluid mechanics

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- Incompressible fluid

Fluid mechanics

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Can we use such equations for blood flow?
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- Incompressible fluid
- Developed flow

Fluid mechanics

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- Incompressible fluid
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- Fixed walls

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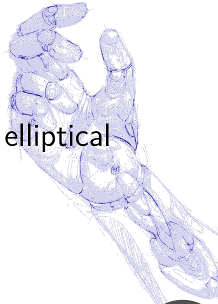


Rise of empirical equations!

Fluid mechanics

Hagen-Poiseuille flow

Considering steady flow:

$$\Delta P = \frac{8\pi\mu L Q}{A^2}$$

Fluid mechanics

Hagen-Poiseuille flow

Considering steady flow:

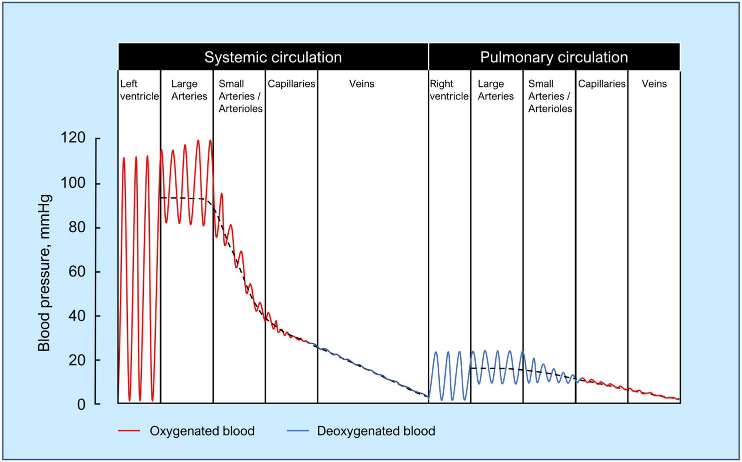
$$\Delta P = \frac{8\pi\mu L Q}{A^2}$$

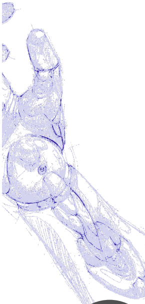
ΔP: Pressure drop
μ: Viscocity
L: Length
Q: Flow rate
A: Crossectional area

Fluid mechanics

Pressure drop

$$\Delta P = \frac{8\pi\mu L Q}{A^2}$$

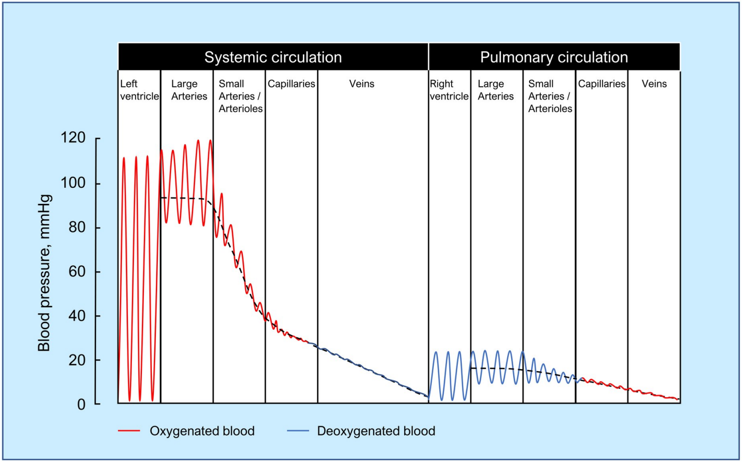




Fluid mechanics

Pressure drop

$$\Delta P = \frac{8\pi\mu L Q}{A^2}$$, A way to calculate pressure along the cardiovascular system



Blood characteristics

Fahraeus-Lindqvist effect

Blood viscosity drops at very small diameters (capilaries)

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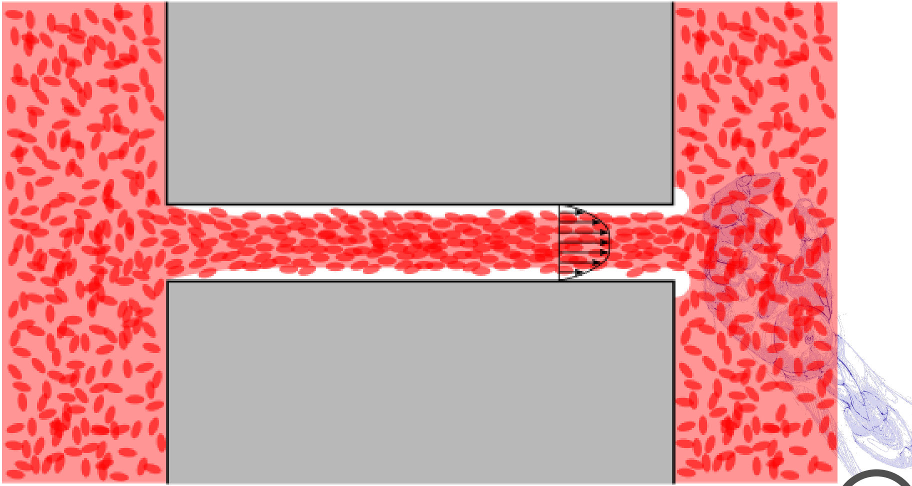

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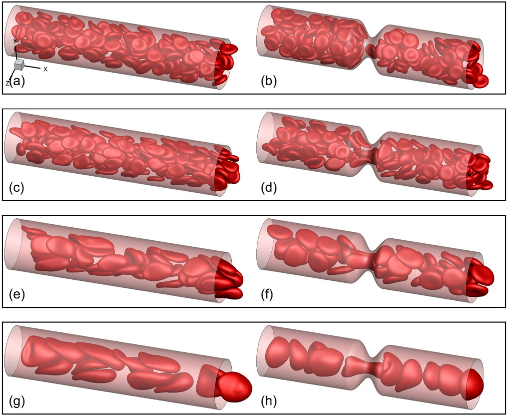
Fahraeus-Lindqvist effect

Blood viscosity drops at very small diameters (capilaries)



Vahidkhah, K. et al. Flow of Red Blood Cells in Stenosed Microvessels. Sci Rep 6, 28194 (2016)

Blood characteristics

Fahraeus-Lindqvist effect

Based on the Hagen-Poiseuille flow equation:

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Blood characteristics

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μ_e: Effective Viscocity
R: Radius ΔP: Pressure drop
Q: Volumetric flow rate
L: Length of capillary

Fluid mechanics

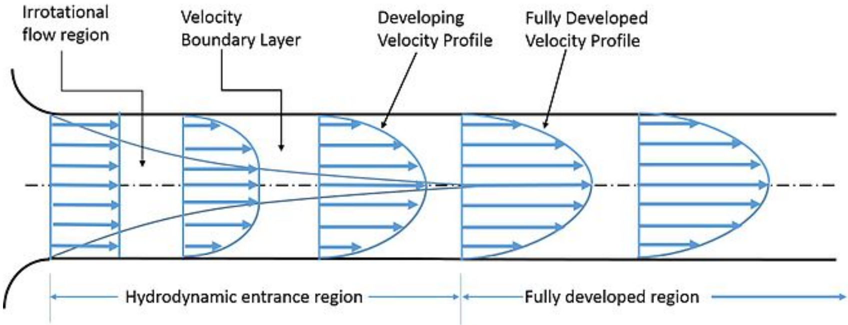
Developed and developing flow

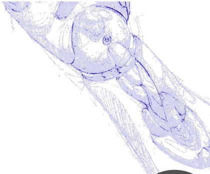
Hagen-Poisseuille can be used for “fully developed flow”

Fluid mechanics

Developed and developing flow

Hagen-Poisseuille can be used for "fully developed flow"





Fluid mechanics

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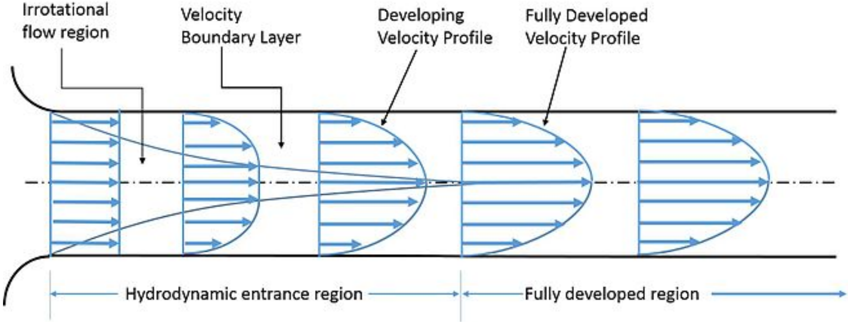
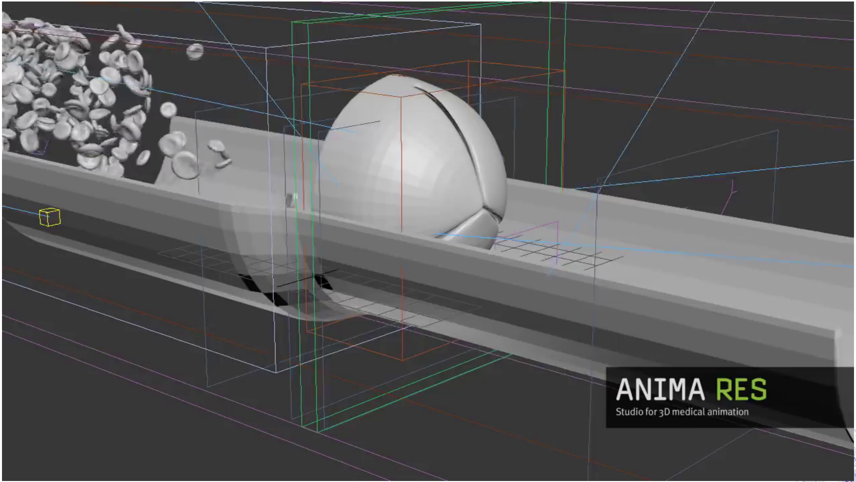


$$\frac{l}{d} = 0.06Re$$, laminar flow and Re > 50

$$\frac{l}{d} = 0.693Re$$, turbulent flow

Fluid mechanics

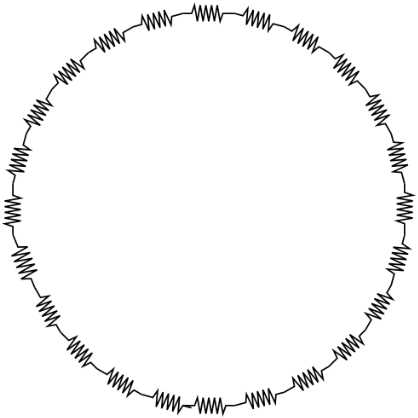
Flow in elastic walls



Anima RES youtube channel

Fluid mechanics

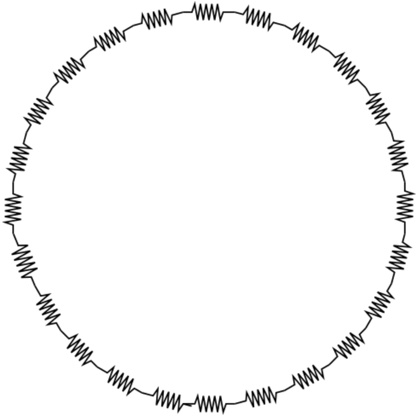
Flow in elastic walls



Fluid mechanics

Flow in elastic walls

$$2\sigma_{\theta\theta}h = \int_0^{\pi} \left(p(x) - p_e\right) r(x) sin\theta d\theta$$



Fluid mechanics

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Fluid mechanics

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Fluid mechanics

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Fluid mechanics

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Results: $\left[1 - \frac{r_0}{Eh}\,(p(x) - p_e)\right]^{-4} dp = -\frac{8\mu}{\pi r_0^4} Q dx$

Fluid mechanics

Flow in elastic walls

$$\left[1 - \frac{r_0}{Eh}\left(p(x) - p_e\right)\right]^{-4} dp = -\frac{8\mu}{\pi r_0^4} Q dx$$

Fluid mechanics

Flow in elastic walls

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Conditions: $P=P_1$ at $x=0$, $P=P_2$ at $x=L$. By integration over the length

Fluid mechanics

Flow in elastic walls

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Fluid mechanics

Flow in elastic walls

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$$Q = \frac{\pi r_0^3 Eh}{24\mu L}\left\{\left[1 - \frac{r_0}{Eh}\left(p_1 - p_e\right)\right]^{-3} - \left[1 - \frac{r_0}{Eh}\left(p_2 - p_e\right)\right]^{-3}\right\}$$

Fluid mechanics

Flow in elastic walls

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Fluid mechanics

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We can calculate pressure drop from Hagen-Poisseuille

Fluid mechanics

Flow in elastic walls

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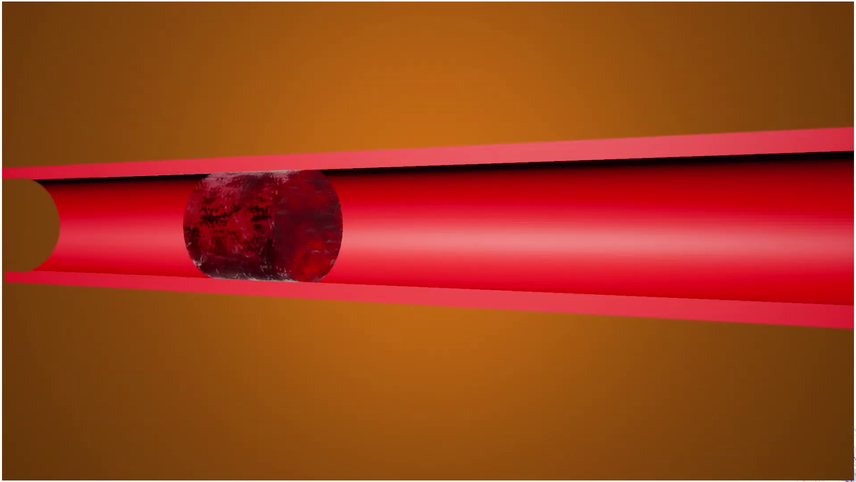
We can calculate pressure drop from Hagen-Poisseuille

What are the assumptions we made?

Fluid mechanics

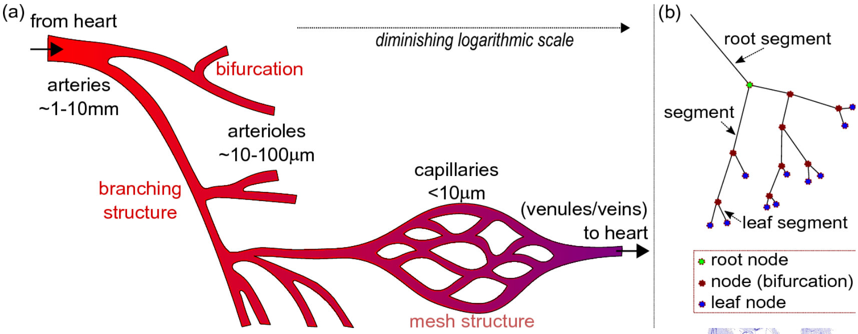
Pulsatile flow

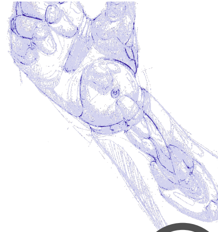


Guelph physics youtube channel

Fluid mechanics

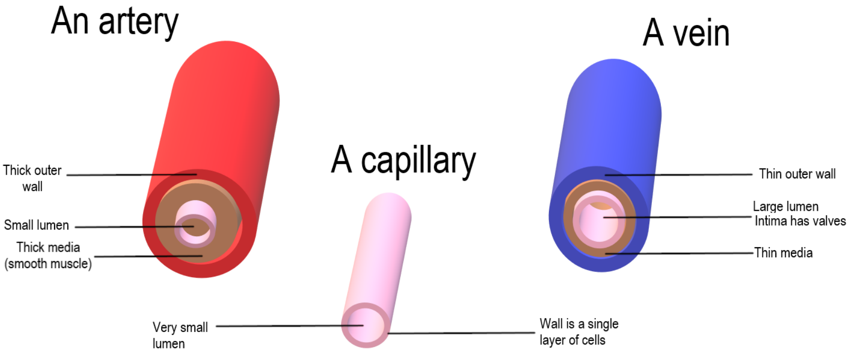
Burifications





Fluid mechanics

Blood vessel structure



Fluid biomechanics

How do we combine everything together?

A lot of complex phenomena, bring the models to its limits.

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Computational Fluid Mechanics

Different approaches

Computational Fluid Mechanics

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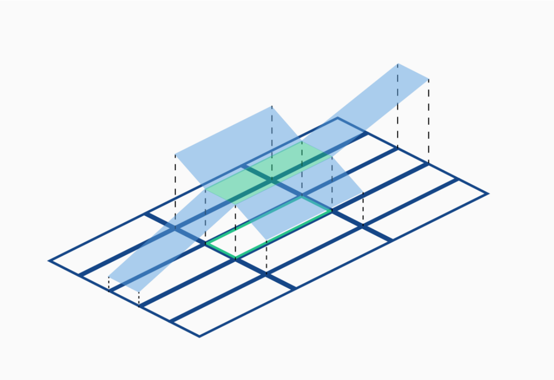
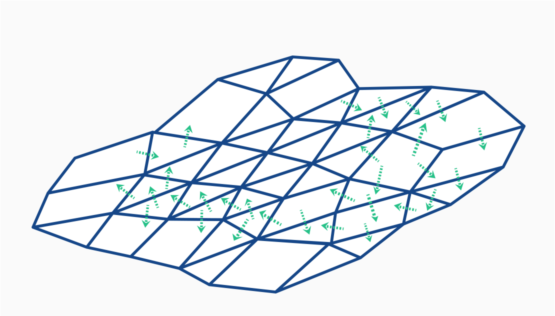


- Finite Differences Method (FDM)

Computational Fluid Mechanics

Different approaches



- Finite Differences Method (FDM)
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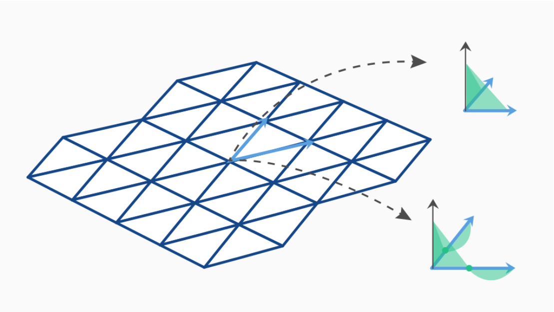
Computational Fluid Mechanics

Different approaches



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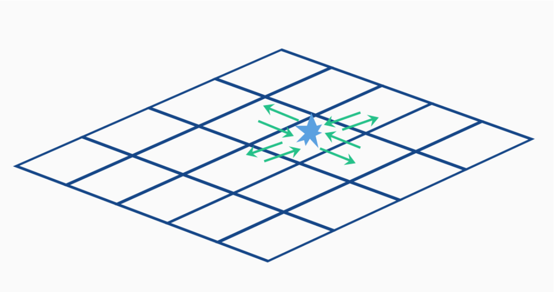
Computational Fluid Mechanics

Different approaches



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Computational Fluid Mechanics

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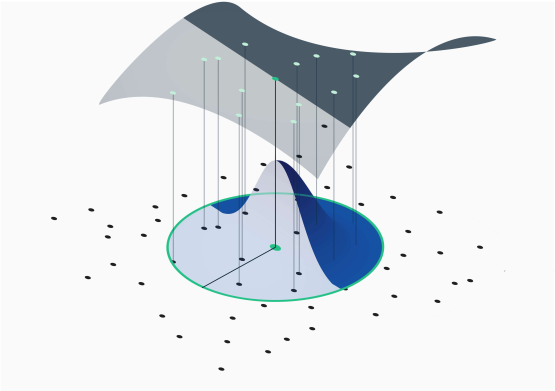

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Computational Fluid Mechanics

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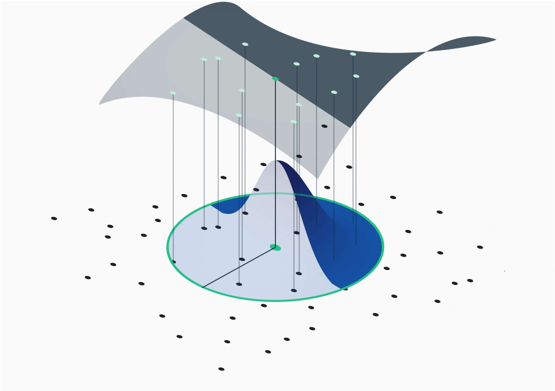

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https://www.dive-solutions.de/blog/cfd-methods

Coming up next

Musculoskeletal modelling

Questions?