Laboratory 2

Direct geometric model

This laboratory presents an algorithm for the determination of the direct geometric model for an 'open chain' manipulation structure. We will:

- Define the homogeneous transformation matrices for the joints of a manipulation structure with "n" degrees of freedom.
- Define and calculate the position vector $Z_p = [x \;\; y \;\; z]^T$ for the specified manipulation structure.

2.1 Anatomy of a robot

The arm of a robot is composed of a succession of rigid segments, called *links*. The links are connected with each other through articulations, called *joints*. The links and joints are forming a kinematic chain. The kinematic chain can be open or closed. Every link in a closed kinematic chain, is connected at least with two joints. In the case of an open kinematic chain, the first like (base) and the last link (end effector), have only a single joint. There are two types of joints available when constructing a robot: joints that allow a rotation (R) and joints that allow a translation (T). If a joint is actuated with the help of a motor, it is called motor joint.

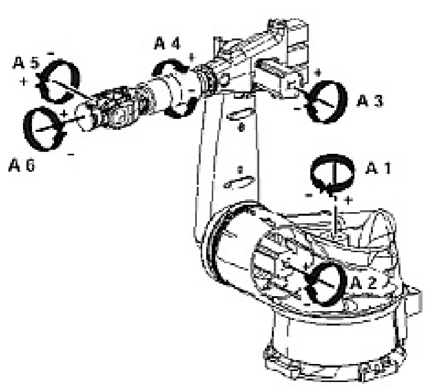

Figure 2.1: The KR 60 P2 Robot

In figure 2.1 the KR 60 P2 robot of KUKA Roboter GmbH is presented. All the joints of this robot are rotational joints (robot RRR/RRR), denoted $A1 - A6$.

The purpose of the direct geometric model is to provide a connection between the movement of a joint (described with a variable for each one of the joints) and the movement of the end effector. With other words, we intend to establish a link between the generalised coordinates of each joint and the Cartesian coordinates of the end effector.

2.2 Direct geometric model of an open chain manipulation structure

For a sequence of bodies **i-j-k**, within a kinematic chain of a industrial robot, the iterative calculation of its position uses the relationship:

$$r_{ik}^{(i)} = r_{ij}^{(i)} + R_{ij} \cdot r_{jk}^{(j)} \tag{2.1}$$

$$R_{ik} = R_{ij} \cdot R_{jk} \tag{2.2}$$

where (**i**) is the index for a reference link towards which the position is referenced.

Vector $r_{ij}^{(i)}$ defines the position of body (j) in respect to body (i), while R_{ij} defines the orientation of body (j) in respect to the body (i) (Direction cosine matrix, of dimension 3×3).

Considering a sequence that includes the base body (denoted "1"):
$1 - j - k$, (see figure 2.2) we can write:

$$r_{1k}^{(1)} = r_{1j}^{(1)} + R_{1j} \cdot r_{jk}^{(j)} \tag{2.3}$$

$$R_{1k} = R_{1j} \cdot R_{jk} \tag{2.4}$$

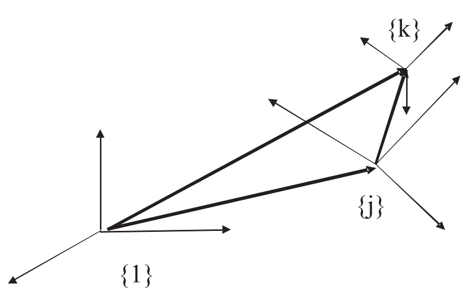

Figure 2.2: Sequence of Cartesian systems and the relationship between them

We can write the relationships (2.1) și (2.2) as:

$$\begin{bmatrix} r_{ik}^{(i)} \\ \\ 1 \end{bmatrix} = \begin{bmatrix} R_{ij} & r_{ij}^{(i)} \\ \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} r_{jk}^{(j)} \\ \\ 1 \end{bmatrix} \tag{2.5}$$

Matrix

$$H_{ij} = \begin{bmatrix} R_{ij} & r_{ij}^{(i)} \\ 0 & 1 \end{bmatrix} \tag{2.6}$$

is a homogeneous transformation matrix, with R_{ij} orientation matrix (Direction Cosine Matrix, with dimension 3×3) and $r_{ij}^{(i)}$ position vector (with dimension 3×1).

The equation (2.3) can be written in the following form:

$$\begin{bmatrix} r_{1k}^{(1)} \\ 1 \end{bmatrix} = \begin{bmatrix} R_{1i} & r_{1i}^{(1)} \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} r_{ik}^{(i)} \\ 1 \end{bmatrix} \tag{2.7}$$

The equation (2.7) together with relationship (2.5) leads to:

$$\begin{bmatrix} r_{1k}^{(1)} \\ 1 \end{bmatrix} = \begin{bmatrix} R_{1i} & r_{1i}^{(1)} \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} R_{ij} & r_{ij}^{(i)} \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} r_{jk}^{(i)} \\ 1 \end{bmatrix} \tag{2.8}$$

Keeping the notation from relationship (2.6) we obtain:

$$\begin{bmatrix} r_{1k}^{(1)} \\ 1 \end{bmatrix} = H_{1i} \cdot H_{ij} \cdot \begin{bmatrix} r_{jk}^{(j)} \\ 1 \end{bmatrix} = H_{1j} \cdot \begin{bmatrix} r_{jk}^{(j)} \\ 1 \end{bmatrix} \tag{2.9}$$

where

$$H_{1j} = H_{1i} \cdot H_{ij} \tag{2.10}$$

For a structure with "n" bodies, the position and orientation of body "n" (end-effector), in respect to the base body 1 (inertial reference), are given by:

$$H_{1n} = H_{12} \cdot H_{23} \cdot .... \cdot H_{n-1,n} \tag{2.11}$$

The homogeneous transformation matrices $H_{i-1,i}$ are referring to the kinematic joints of the structure (links *i-1*,i) which are rotational or translational. The definition the matrices for these situation, is found in relationships (1.3 - 1.8).

The algorithm for the determination of the direct geometric model, could be the following:

1. We attach a variable on each motor joint (generalised coordinate) $q_1 ... q_n$.
2. We attach a coordinate system on each link, starting with the base (1) until the end effector (n).

3. We calculate iteratively, the position and orientation of each body in respect to the base body, using relationships 2.3 and 2.4. Alternatively we can express this with homogeneous matrices, from relationship 2.11

For example, for the structure of the RTT robot of figure 2.4, homogeneous elementary transformation matrices, are:

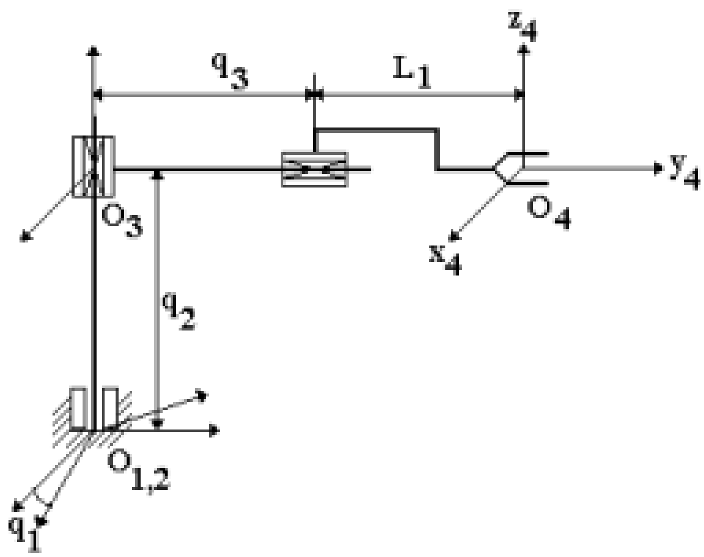

Figure 2.4: Robot RTT

$$T_{12} = \begin{bmatrix} \cos(q_1) & -\sin(q_1) & 0 & 0 \\ \sin(q_1) & \cos(q_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.12}$$

$$T_{23} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.13}$$

$$T_{34} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & q_3 + L_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.14}$$

For efficiency reasons, we will use the following notation[1]:

$$\begin{cases} \cos(q_i) \overset{\text{not}}{=} c_i \\ \\ \sin(q_i) \overset{\text{not}}{=} s_i \end{cases} \quad \textit{pentru orice coordonată generalizată } q_i,\ i = \overline{1..n} \tag{2.15}$$

[1]To each robot structure having **n** motor joints, we can attach **n** generalised coordinates (robot coordinates) $q_1,\ q_2,\ \ldots\ q_n$.

In this condition, the matrix T_{12} becomes:

$$T_{12} = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.16}$$

We determine the direct geometric model using post-multiplication of the elementary matrices that characterise the structure, starting from the base body until the end-effector:

$$T = T_{14} = T_{12} \cdot T_{23} \cdot T_{34} \tag{2.17}$$

$$T_{14} = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & q_3 + L_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.18}$$

resulting in direct geometric model in the following form:

$$T_{14} = \begin{bmatrix} c_1 & -s_1 & 0 & -(q_3 + L_1) \cdot s_1 \\ s_1 & c_1 & 0 & (q_3 + L_1) \cdot c_1 \\ 0 & 0 & 1 & q_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.19}$$

The position parameters of the end-effector are given in the final column of matrix T_{14}:

$$Z_P = \begin{cases} x_{ef} &= -(q_3 + L_1) \cdot \sin(q_1) \\ y_{ef} &= (q_3 + L_1) \cdot \cos(q_1) \\ z_{ef} &= q_2 \end{cases} \tag{2.20}$$

2.3 Proposed problems

1. We consider the robotic structure from figure 2.5:

a) Determine the direct geometric model using iterative relationships 2.3 and 2.4.

b) Determine the direct geometric model using homogeneous transformation matrices.

c) Calculate the position obtained by imposing the following position of joints:

$$\begin{cases} q_1 &= 0 \\ q_2 &= -\frac{\pi}{20} \\ q_3 &= \frac{\pi}{20} \\ q_4 &= 10 \end{cases}$$

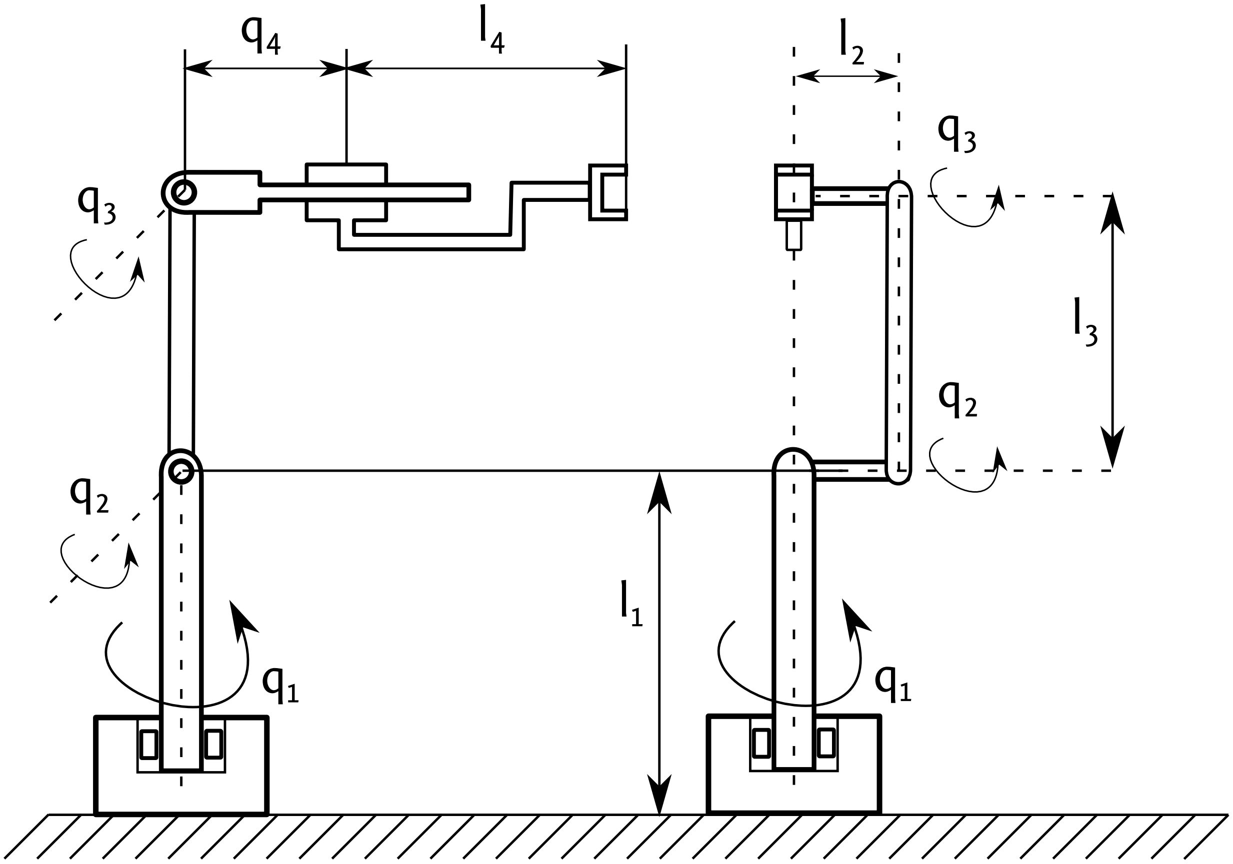

Figure 2.5: Robot RRRT