Forward kinematics

Joints, links, degrees of freedom

TECHNICAL UNIVERSITY
OF CLUJ-NAPOCA
ROMANIA

Last update: October 7, 2024

Agenda

- Robot modeling
- Degrees of freedom
- Work envelope
- Joint types
- Links
- Forward kinematics

Recap

What we saw last week

$$Trans(X, a) = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Trans(Y, b) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Trans(Z, c) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Recap

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$$RotX(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cos\theta & -sin\theta & 0 \\ 0 & sin\theta & cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$RotY(\phi) = \begin{bmatrix} cos\phi & 0 & sin\phi & 0 \\ 0 & 1 & 0 & 0 \\ -sin\phi & 0 & cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

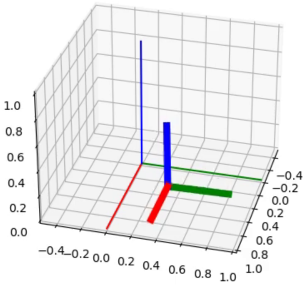
$$RotZ(\omega) = \begin{bmatrix} cos\omega & -sin\omega & 0 & 0 \\ sin\omega & cos\omega & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Recap

What we saw last week

Multiplication from the left results in transformation according to the axes of the base coordinate frame.

$$V'_O = RotX(\theta)V_O$$

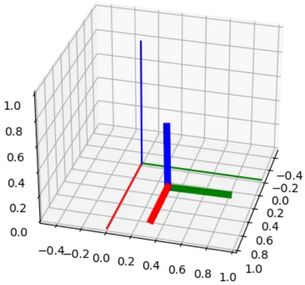


Recap

What we saw last week

Multiplication from the right results in transformation according to the axes of the transformed coordinate frame.

$$V_O' = V_O Rot X(\theta)$$

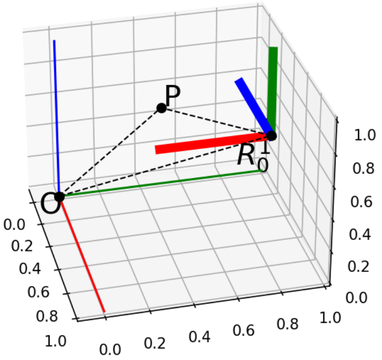


Recap

What we saw last week

Sometimes, we know the coordinates of a point in one coordinate frame, but we need to describe it in a second frame. This is possible if we know the relative position of the two coordinate frames

$$P_0 = R_0^1 P_1$$

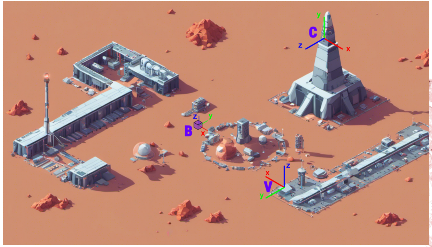


Recap

What we saw last week

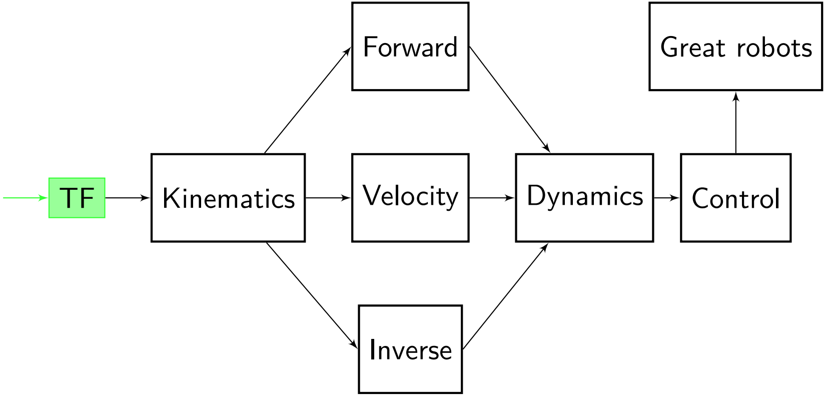
We can also calculate the transformation between two frames, if we know they transformation w.r.t. a third base frame

$$R_v^b = R_v^c R_c^b$$

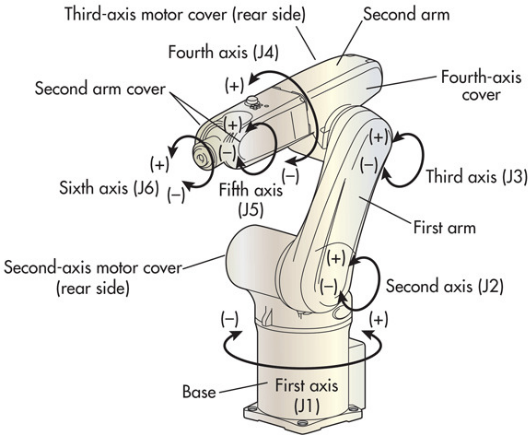


Grand scheme

The big picture

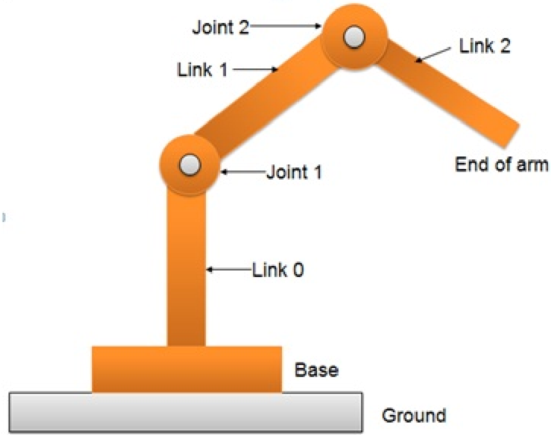


In this course we will mainly talk about stationary articulated robots



Links

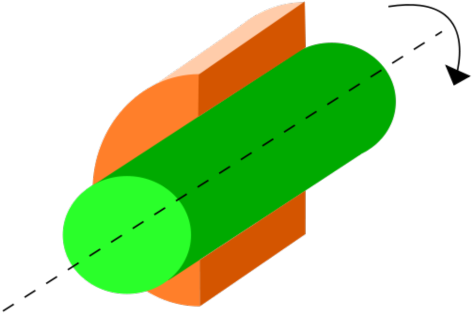
A link is a structural part of any robot. It is usually stiff and is modelled as a rigid (non-deformable) part. They can have any shape, according to the design of the robot.



Joints

Revolute joints

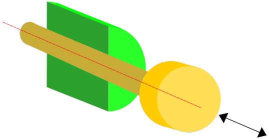
A revolute joint is a joint that allows motion that changes the orientation of a segment by rotating around a fixed axis. They can add one degree of freedom to a robot.



Joints

Prismatic joints

A prismatic joint is a joint that allows motion that changes the position of a segment by translating along an axis. They can add one degree of freedom to a robot.



Robot modeling

Degrees of freedom

When talking about degrees of freedom for a robot we refer to the freedom of the robot to vary **independently** the position and orientation of its end-effector in different directions.

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E.g. a planar robot with two revolute joints and two links has two degrees of freedom as it can vary **independently** the position of the end-effector in the x and y direction.

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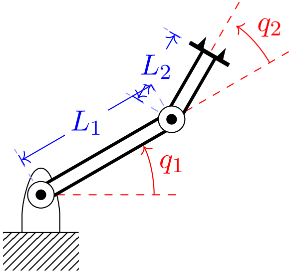

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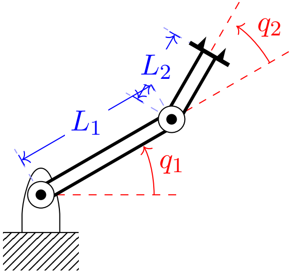
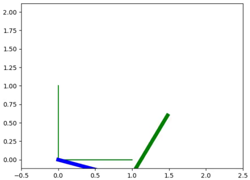
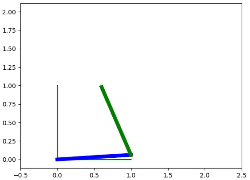
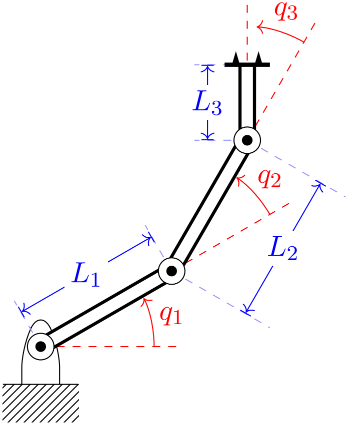






If we add one more joint, we add another degree of freedom



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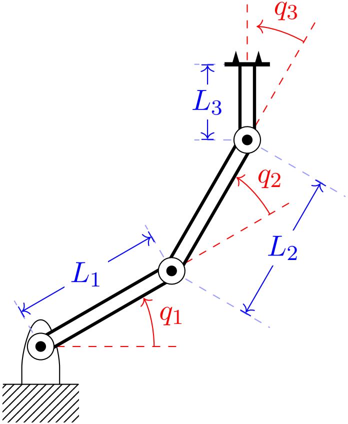

This manipulator can independently vary the x and y position of the end effector, while also controlling the orientation (3 degrees of freedom).

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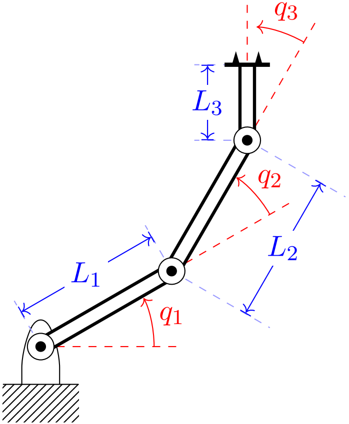

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What happens if we add one more joint?

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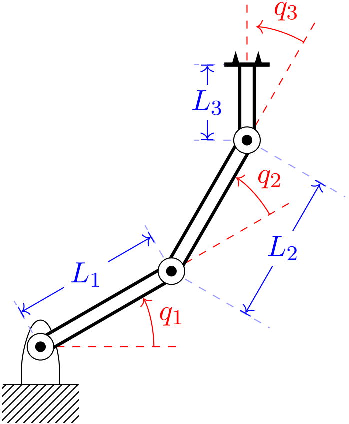


This manipulator can independently vary the x and y position of the end effector, while also controlling the orientation (3 degrees of freedom).

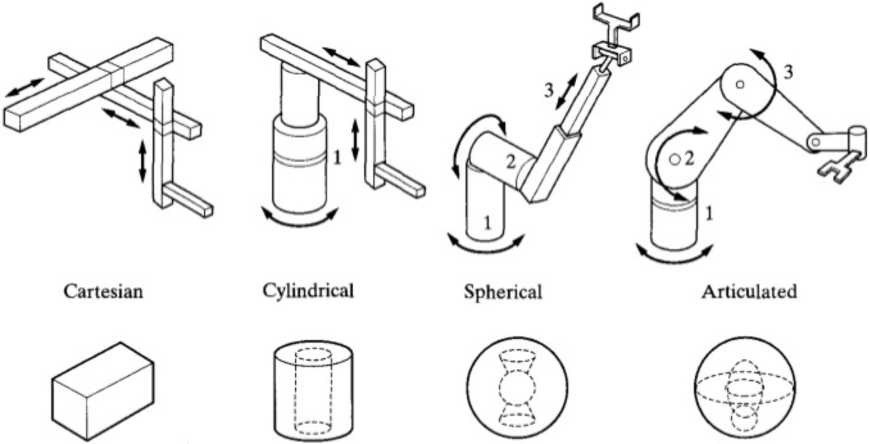
What happens if we add one more joint?

$$M = 6(N-1) + \sum_{i=1}^{j} c_i$$

Robot modeling

Work envelope

The combination of links and joints defines the degrees of freedom to a robot. Besides that, it also defines the work envelope of the robot.



Robot modeling

Work envelope

Even if a robot has a certain number of degrees of freedom, some of them might be lost in some areas of the work envelope.

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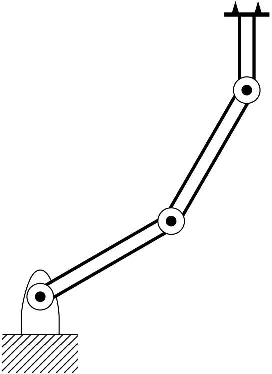

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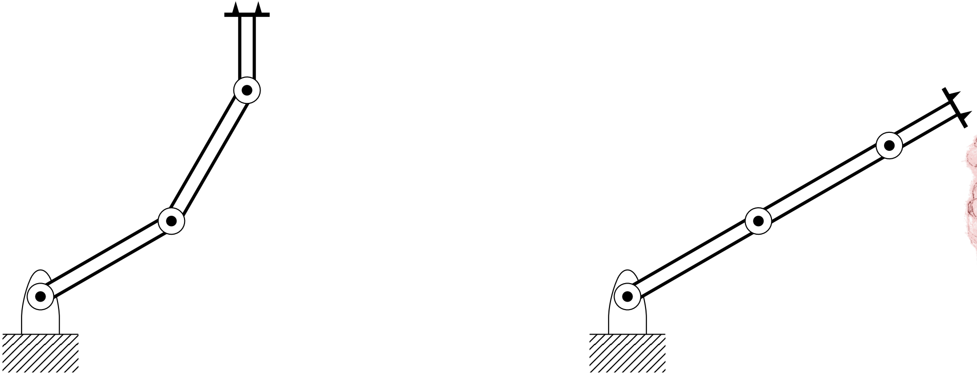

Forward kinematics

Definition

The forward kinematics model (FKM) is a mathematical tool that allows us to calculate the position and orientation (pose) of a robot's point of interest if we know the state of the joints and the lengths of the links.

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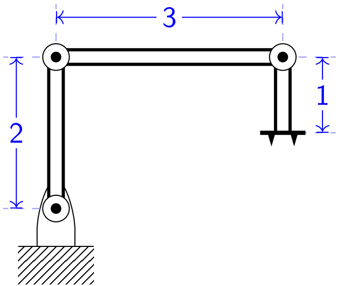
In simple words

How do I calculate the pose of the end-effector if I know the joint angles (coordinates)?

Forward kinematics

Definition



How do we calculate the pose of the end-effector of this robot?

Forward kinematics

Definition

We describe the pose of the end-effector using a 4x4 transformation matrix (contains information about position and orientation).

$$T = \left[\begin{array}{ccc|c} 3 & \times & 3 & 3\times 1 \\ \hline 1 & \times & 3 & 1\times 1 \end{array}\right] = \left[\begin{array}{ccc|c} & rotation & & \begin{array}{c} trans- \\ la- \\ tion \end{array} \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \tag{1}$$

Forward kinematics

Calculation

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- We attach a fixed coordinate frame on the base of the robot.
- We attach a coordinate frame on each link at their joints.

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- We calculate the transformation between each subsequent coordinate frame.

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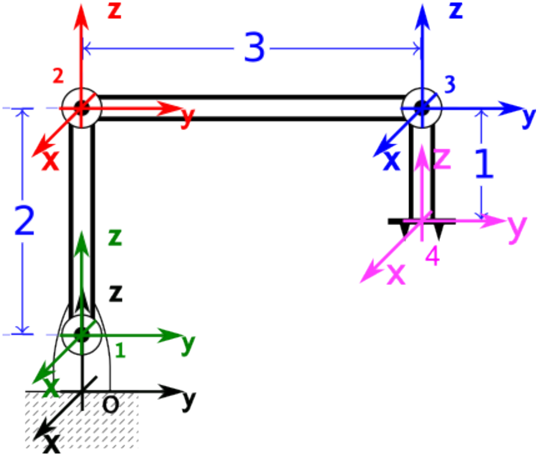
- We attach a fixed coordinate frame on the base of the robot.
- We attach a coordinate frame on each link at their joints.
- We attach a coordinate frame at the end-effector.
- We calculate the transformation between each subsequent coordinate frame.
- We combine the transformations to calculate the overall transformation from base to end-effector.

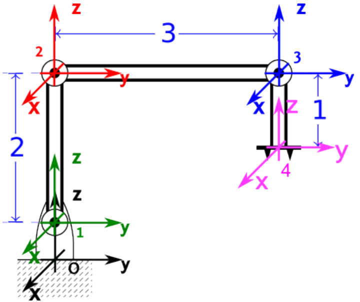
Forward kinematics

Calculation



Forward kinematics

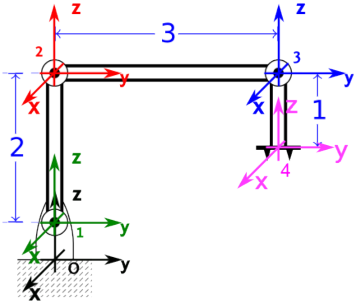
Static calculation



$$R_0^1 = Trans(Z,1) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R_1^2 = Trans(Z,2) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Forward kinematics

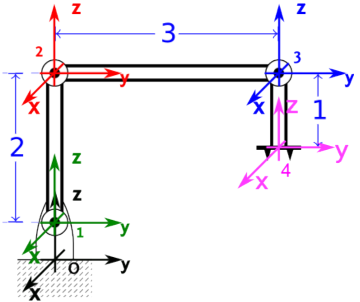
Static calculation



$$R_2^3 = Trans(Y,3) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R_3^4 =$$

Forward kinematics

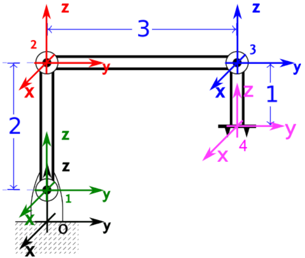
Static calculation



$$R_2^3 = Trans(Y,3) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R_3^4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Forward kinematics

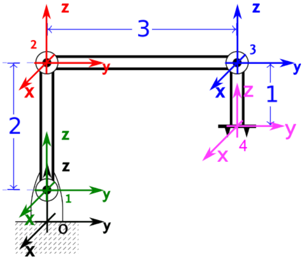
Static calculation



$$R_0^4 =$$

Forward kinematics

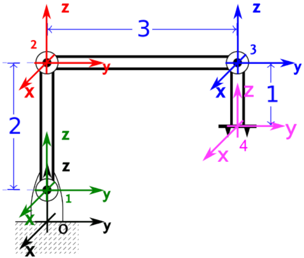
Static calculation



$$R_0^4 = R_0^1 R_1^2 R_2^3 R_3^4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Forward kinematics

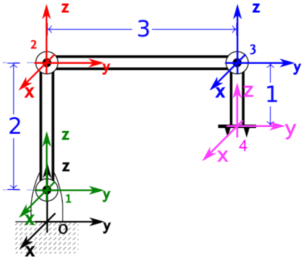
Static calculation



$$R_0^4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} & & & trans- \\ & rotation & & la- \\ & & & tion \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Forward kinematics

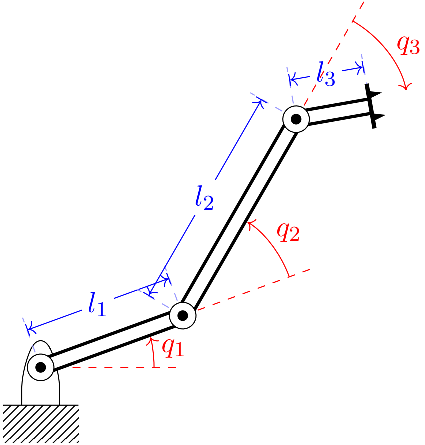
Static calculation



$$R_0^4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} & & & trans- \\ & rotation & & la- \\ & & & tion \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R_m^n = \begin{bmatrix} X_x & Y_x & Z_x & P_x \\ X_y & Y_y & Z_y & P_y \\ X_z & Y_z & Z_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

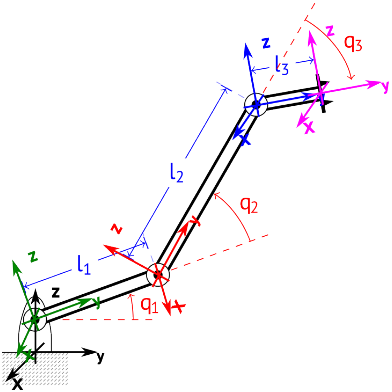
Forward kinematics

What about other configurations?



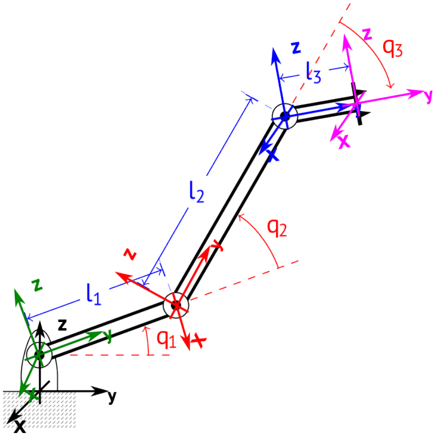
Forward kinematics

Dynamic calculation



Forward kinematics

Dynamic calculation

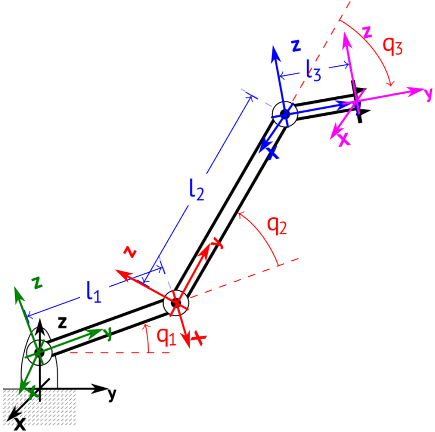


$$R_0^1 = Trans(Z,1)R(X,q_1) =$$
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_1 & -s_1 & 0 \\ 0 & s_1 & c_1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_1^2 = Trans(Y,l_1)R(X,q_2) =$$
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & l_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_2 & -s_2 & 0 \\ 0 & s_2 & c_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Forward kinematics

Dynamic calculation



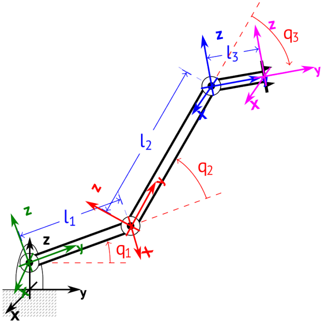
$$R_2^3 = Trans(Y, l_2)R(X, q_3) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & l_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_3 & -s_3 & 0 \\ 0 & s_3 & c_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_3^4 = Trans(Y, l_3) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & l_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_0^4 = R_0^1 R_1^2 R_2^3 R_3^4$$

Forward kinematics

Dynamic calculation



$$R_0^4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{1,2,3} & -s_{1,2,3} & l_3 c_{1,2,3} + l_2 c_{1,2} + l_1 c_1 \\ 0 & s_{1,2,3} & c_{1,2,3} & l_3 s_{1,2,3} + l_2 s_{1,2} + l_1 s_1 + 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Forward kinematics

Dynamic calculation

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Forward kinematics

The FKM is a transformation matrix, a function of the joint positions and link lengths. If we know these variables, we can calculate the position and orientation of the end effector (or any other point).

Forward kinematics

Food for thought

Why do we multiply from the right each transformation?

Forward kinematics

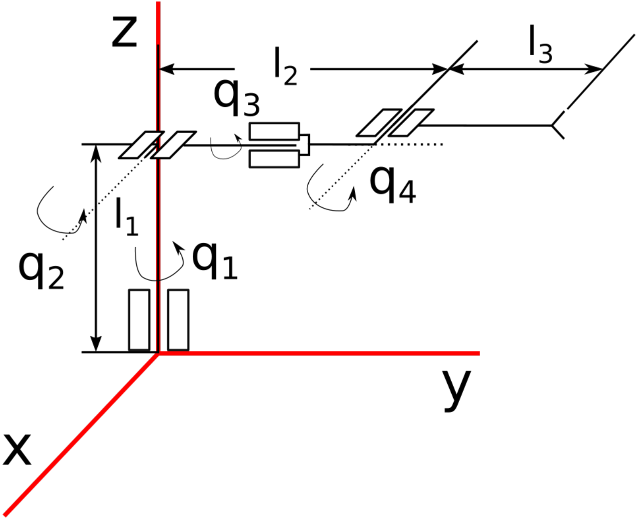
Food for thought

Why do we multiply from the right each transformation?

Could we figure out how to multiply from the left?

Forward kinematics

Example in 3D



Forward kinematics

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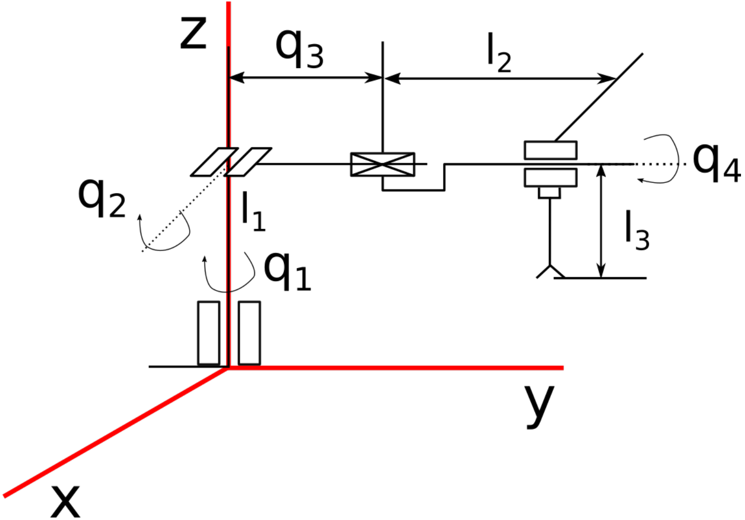
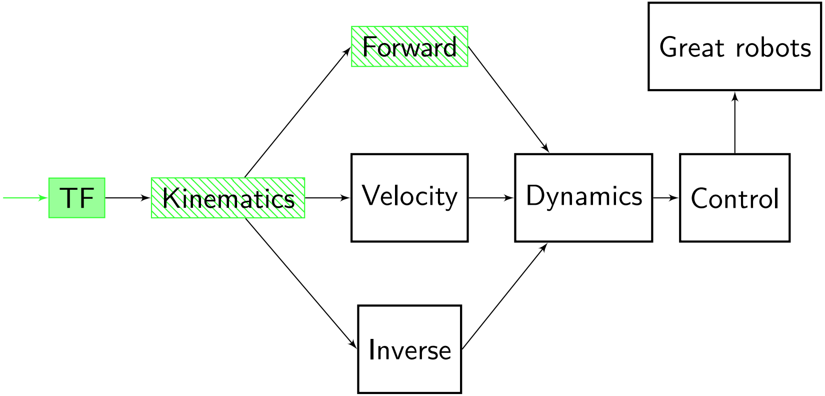


Grand scheme

The big picture



Questions?