Denavit-Hartenberg convention

Why, what, how

TECHNICAL UNIVERSITY OF CLUJ-NAPOCA ROMANIA

Last update: October 17, 2023

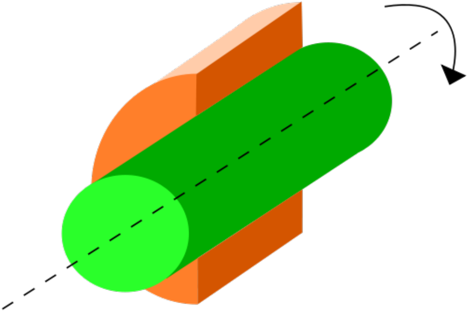
Agenda

- Background
- Drawing robots in 3D
- Defining DH parameters
- Examples, examples, examples
- Modified DH parameters

Recap

What we saw last week

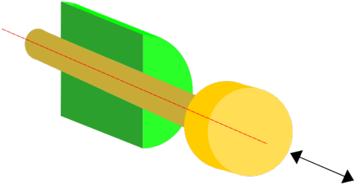
A revolute joint is a joint that allows motion that changes the orientation of a segment by rotating around a fixed axis. They can add one degree of freedom to a robot.



Recap

What we saw last week

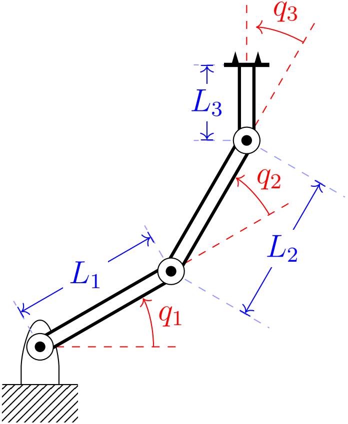
A prismatic joint is a joint that allows motion that changes the position of a segment by translating along an axis. They can add one degree of freedom to a robot.



Recap

What we saw last week

If we add one more joint, we add another degree of freedom



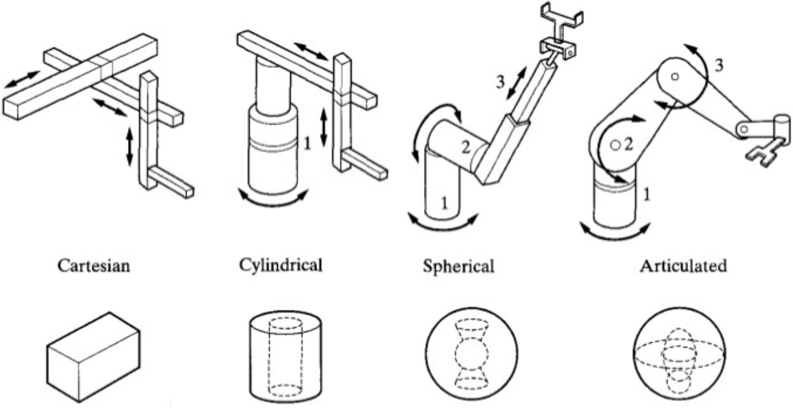
This manipulator can independently vary the x and y position of the end effector, while also controlling the orientation (3 degrees of freedom).

What happens if we add one more joint?

Recap

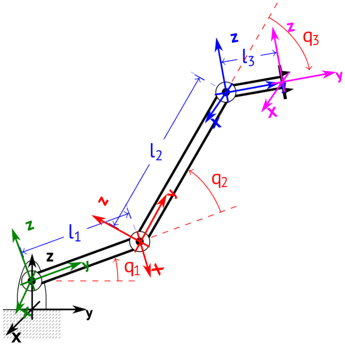
What we saw last week

The combination of links and joints defines the degrees of freedom to a robot. Besides that, it also defines the work envelope of the robot.



Recap

What we saw last week



$$R_0^4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{1,2,3} & -s_{1,2,3} & l_3 c_{1,2,3} + l_2 c_{1,2} + l_1 c_1 \\ 0 & s_{1,2,3} & c_{1,2,3} & l_3 s_{1,2,3} + l_2 s_{1,2} + l_1 s_1 + 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Recap

What we saw last week

$$R_0^4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{1,2,3} & -s_{1,2,3} & l_2 c_{1,2} + l_3 c_{1,2,3} + l_1 c_1 \\ 0 & s_{1,2,3} & c_{1,2,3} & l_2 s_{1,2} + l_3 s_{1,2,3} + l_1 s_1 + 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Forward kinematics

The FKM is a transformation matrix, a function of the joint positions and link lengths. If we know these variables, we can calculate the position and orientation of the end effector (or any other point).

Denavit-Hartenberg convention

Background

- Introduced by Jacques Denavit and Richard S. Hartenberg

Denavit-Hartenberg convention

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- Describes each link using only four parameters

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- Describes each link using only four parameters
- Can be used for any kinematic chain

Denavit-Hartenberg convention

Background

- Introduced by Jacques Denavit and Richard S. Hartenberg
- Describes each link using only four parameters
- Can be used for any kinematic chain
- Results in a forward kinematics model

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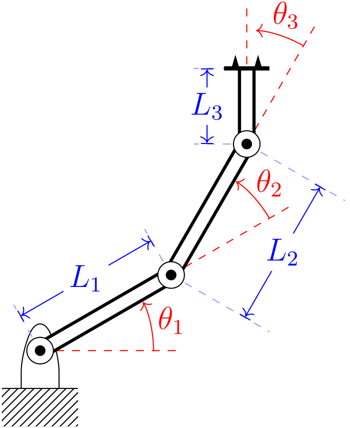
Why?



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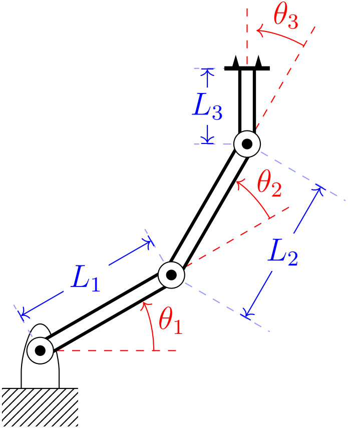
Why?



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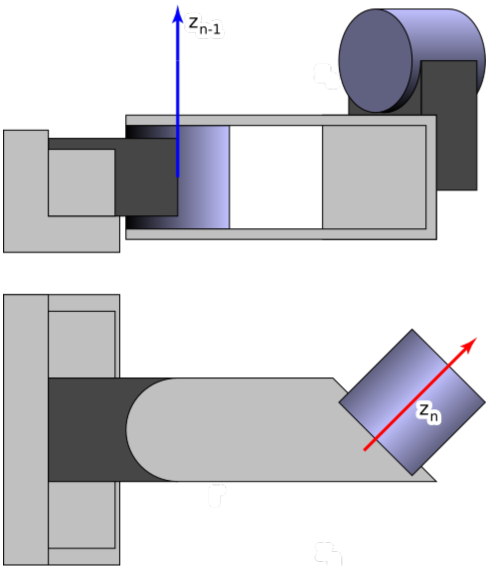
Why?



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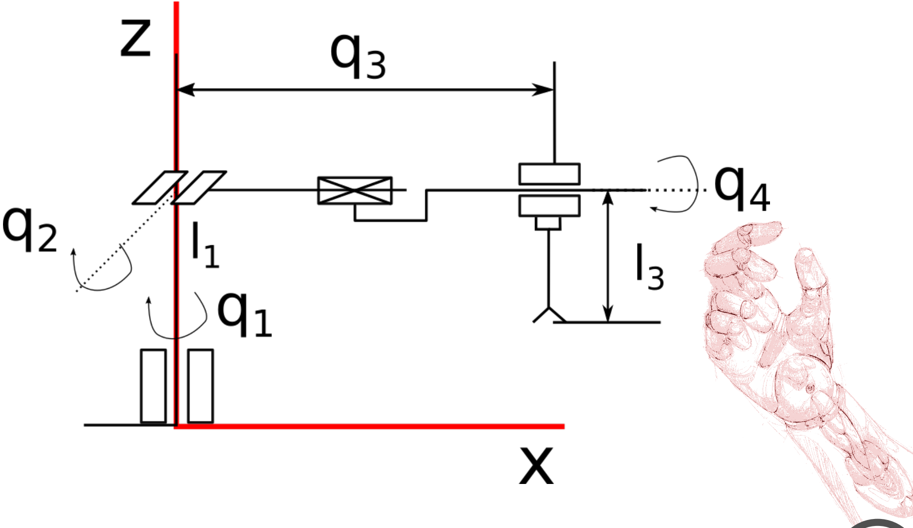
3D robots



Denavit-Hartenberg convention

Definition

DH Parameters

Using the convention, we define four parameters for each link. Two parameters refer to angles and two refer to lengths.

- d: Joint offset (length)
- θ: Joint angle
- r: Link length
- α: Link twist (angle)

Denavit-Hartenberg convention

Definition

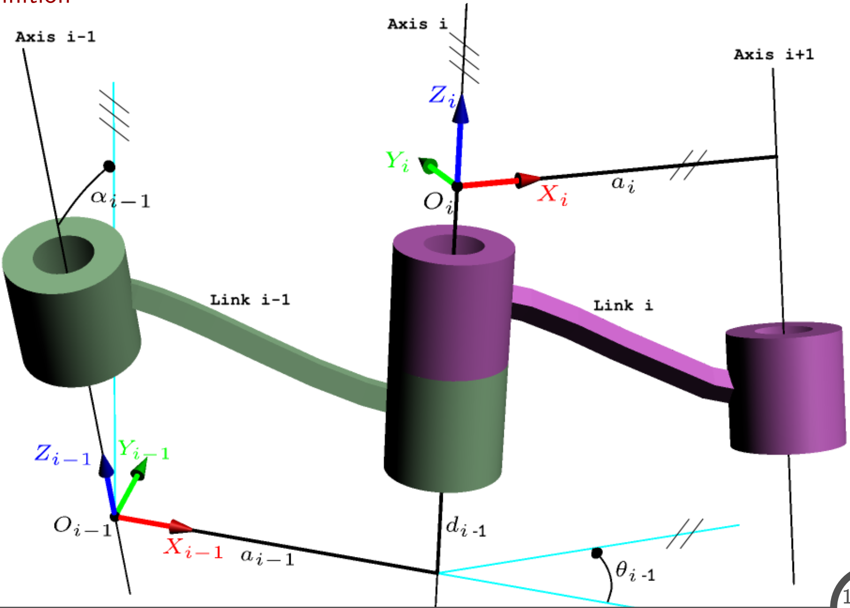
DH Modified Parameters

We define each parameter for the length and angles from joint i until the joint $i+1$

- d_i: Joint offset (length) from joint i to joint i+1
- θ_i: Joint angle from joint i to joint i+1
- r_i: Link length from joint i to joint i+1
- α_i: Link twist (angle) from joint i to joint i+1

Denavit-Hartenberg convention

Definition



Denavit-Hartenberg convention

Definition

DH Parameters

We define four transformation matrices for the transformation from joint i to joint i+1. Two are rotation and two are translation matrices.

$$T_i^{i+1} = [X_i] * [Z_i]$$

Denavit-Hartenberg convention

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$$T_i^{i+1} = [X_i] * [Z_i]$$

where

$[X_i] = Rx(\alpha_i) * Tx(r_i)$

Denavit-Hartenberg convention

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where

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Denavit-Hartenberg convention

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where

$$[X_i] = Rx(\alpha_i) * Tx(r_i)$$

and

$$[Z_i] = Rz(\theta_i) * Tz(d_i)$$

therefore

$$T_i^{i+1} = Rx(\alpha_i) * Tx(r_i) * Rz(\theta_i) * Tz(d_i)$$

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Definition

$$T_i^{i+1} = \left[\begin{array}{ccc|c} \cos\theta_i & -\sin\theta_i & 0 & r_i \\ \sin\theta_i\cos\alpha_i & \cos\theta_i\cos\alpha_i & -\sin\alpha_i & -d_i\sin\alpha_i \\ \sin\theta_i\sin\alpha_i & \cos\theta_i\sin\alpha_i & \cos\alpha_i & d_i\cos\alpha_i \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$T_0^n = T_0^1 * T_1^2 * \ldots * T_{n-1}^n$$

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Calculating the parameters

To calculate the 4 parameters, we first construct coordinate frames (CF) for each joint using the following procedure:

- We align the z-axis of each CF with the axis of rotation/translation of each joint

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Calculating the parameters

To calculate the 4 parameters, we first construct coordinate frames (CF) for each joint using the following procedure:

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- We identify the common perpendicular between subsequent z-axes

Denavit-Hartenberg convention

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- We align X_i with the common perpendiculars between Z_i and Z_{i+1}
- The positive direction for X_i is from Z_i to Z_{i+1}

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Calculating the parameters

Once we have constructed the CFs, we identify the four parameters as following:

- r_i: distance between axes Z_i and Z_{i+1}, measured on axis X_i

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- d_i: distance between axes X_i and X_{i+1}, measured on axis Z_{i+1}

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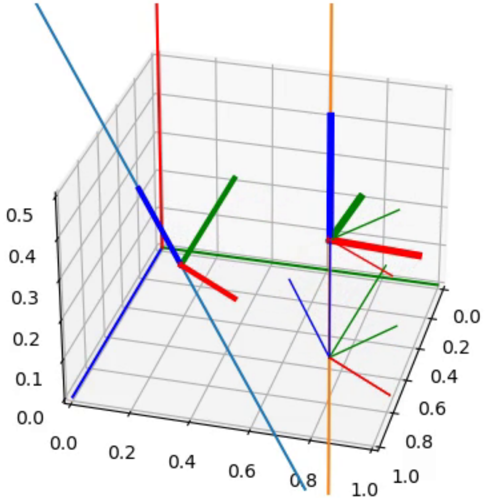
- α_i: angle between axes Z_i and Z_{i+1}, measured around axis X_i
- d_i: distance between axes X_i and X_{i+1}, measured on axis Z_{i+1}
- θ_i: angle between axes X_i and X_{i+1}, measured around axis Z_{i+1}

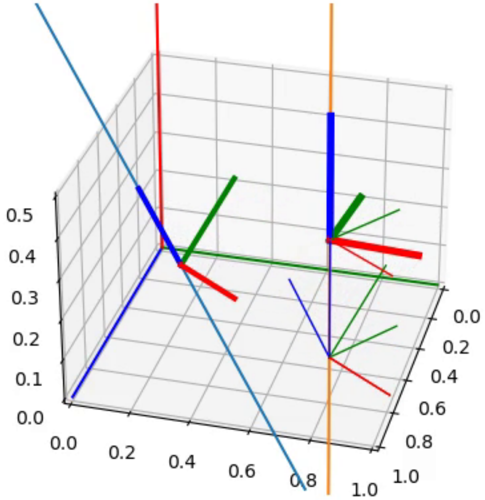
Denavit-Hartenberg convention

Visualising the angles



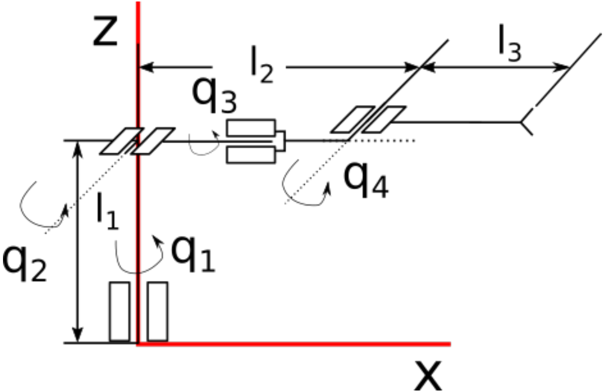
Denavit-Hartenberg convention

Visualising the angles



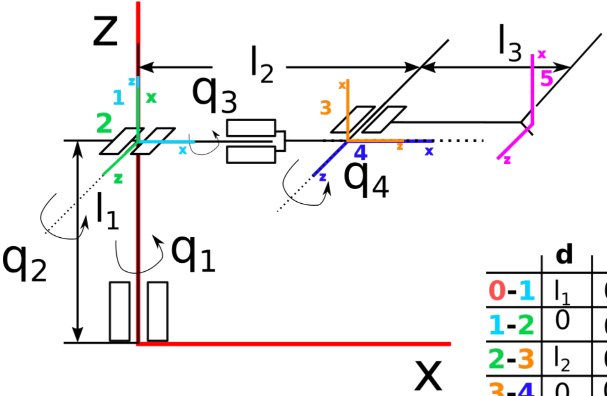
Denavit-Hartenberg convention

Examples



Denavit-Hartenberg convention

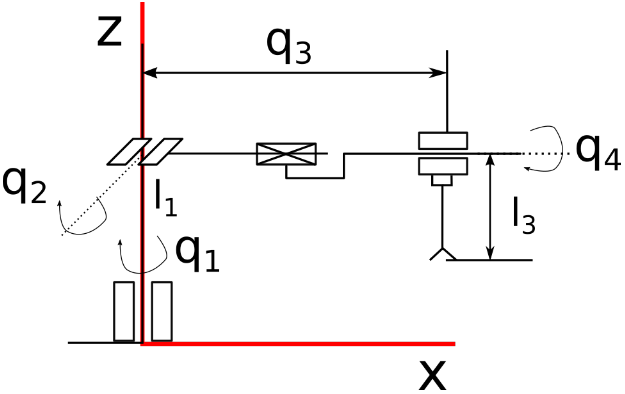
Examples



	d	r	α	θ
0-1	l_1	0	0	q_1
1-2	0	0	$\pi/2$	q_2
2-3	l_2	0	$\pi/2$	q_3
3-4	0	0	$-\pi/2$	q_4
4-5	0	l_3	0	$\pi/2$

Denavit-Hartenberg convention

Examples



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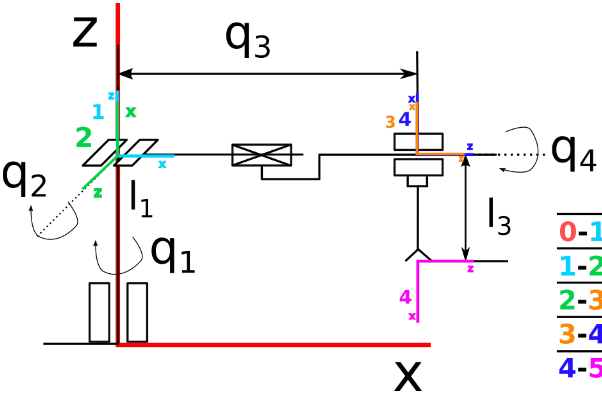
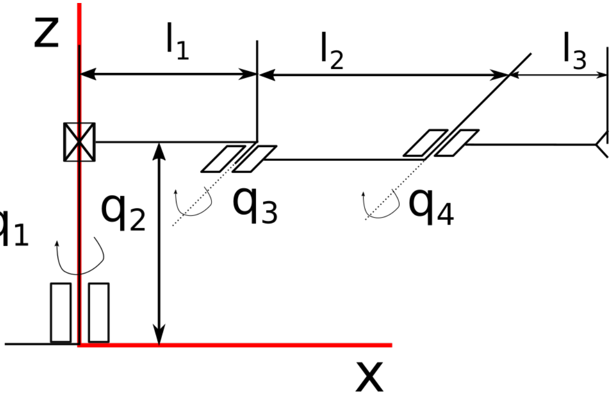
Examples



	d	r	α	θ
0-1	l_1	0	0	q_1
1-2	0	0	$\pi/2$	q_2
2-3	q_3	0	$\pi/2$	0
3-4	0	0	0	q_4
4-5	0	$-l_3$	0	π

Denavit-Hartenberg convention

Examples



Denavit-Hartenberg convention

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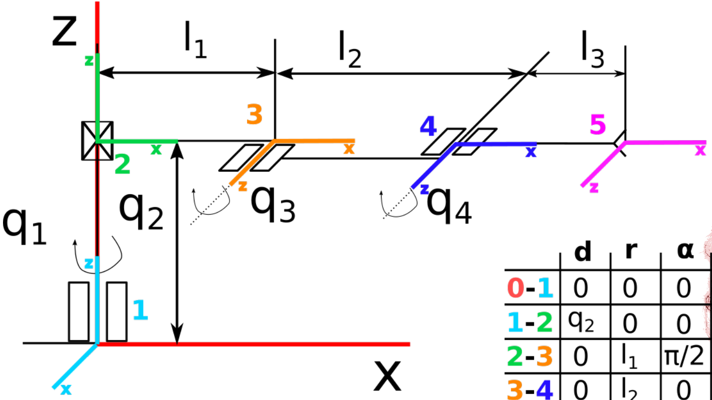
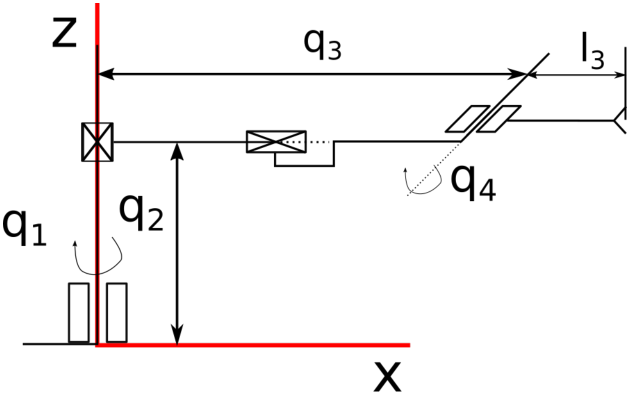


	d	r	α	θ
0-1	0	0	0	q_1
1-2	q_2	0	0	$\pi/2$
2-3	0	l_1	$\pi/2$	q_3
3-4	0	l_2	0	q_4
4-5	0	l_3	0	0

Denavit-Hartenberg convention

Examples



Denavit-Hartenberg convention

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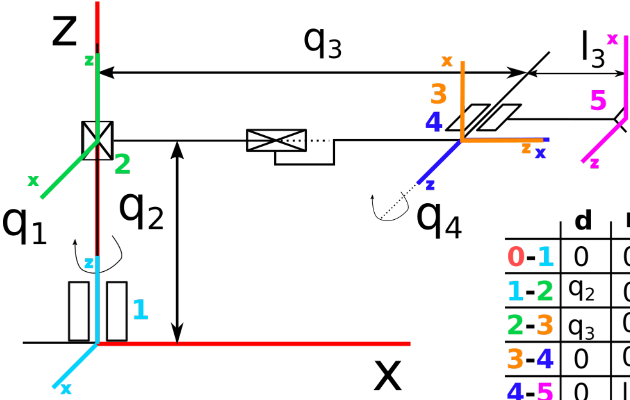


	d	r	α	θ
0-1	0	0	0	q_1
1-2	q_2	0	0	0
2-3	q_3	0	$-\pi/2$	$-\pi/2$
3-4	0	0	$-\pi/2$	q_4
4-5	0	l_3	0	$\pi/2$

Denavit-Hartenberg convention

Parameters θ and d

Revolute joints

If $i + 1$ is a revolute joint, parameter θ_i is always variable and relates to joint variable q_{i+1}

Denavit-Hartenberg convention

Parameters θ and d

Revolute joints

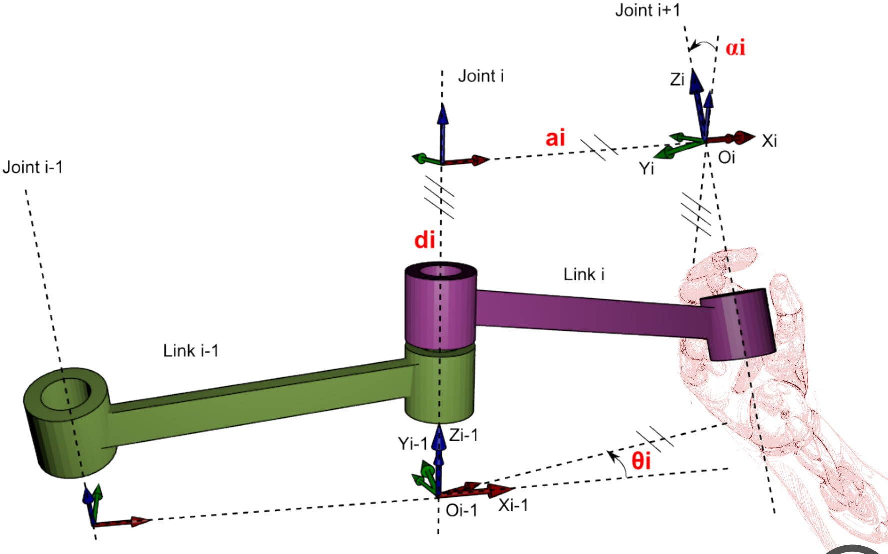
If $i+1$ is a revolute joint, parameter θ_i is always variable and relates to joint variable q_{i+1}

Prismatic joints

If $i+1$ is a prismatic joint, parameter d_i is always variable and relates to joint variable q_{i+1}

Denavit-Hartenberg convention

Alternative form



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- d_i: distance between axes X_{i-1} and X_i, measured on axis Z_{i-1};
- θ_i: angle between axes X_{i-1} and X_i, measured around axis Z_{i-1}.

Denavit-Hartenberg convention

Alternative form

$$T_i^{i+1} = [Z_i] * [X_i]$$

where

$$[Z_i] = Tz(d_i) * Rz(\theta_i)$$

and

$$[X_i] = Tx(r_i) * Rx(\alpha_i)$$

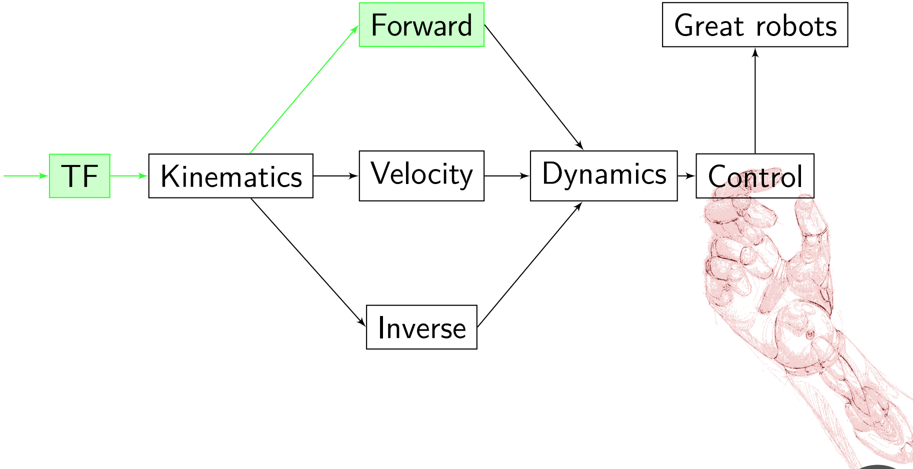
therefore

$$T_i^{i+1} = Tz(d_i) * Rz(\theta_i) * Tx(r_i) * Rx(\alpha_i)$$

$$T_i^{i+1} = \left[\begin{array}{ccc|c} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & r_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & r_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

Grand scheme

The big picture



Questions?