Wheeled mobile robots

Description, Kinematics, Modeling

TECHNICAL UNIVERSITY
OF CLUJ-NAPOCA
ROMANIA

December 16, 2024

Agenda

- Types of wheels and wheeled robots
- Moving around
- Kinematics, modeling

Why wheeled robots?

Why are wheeled robots useful?
Provide some examples of applications

Wheels

Types of wheels

Wheels

Types of wheels

- Fixed wheel

Wheels

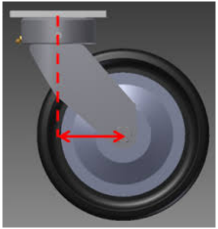
Types of wheels

- Fixed wheel
- Centered wheel

Wheels

Types of wheels

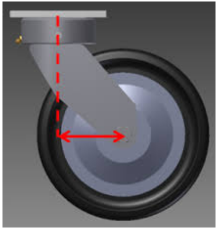
- Fixed wheel
- Centered wheel
- Off-centered wheel



Wheels

Types of wheels



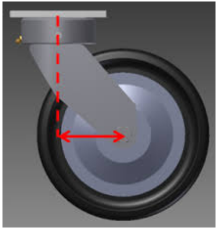



- Fixed wheel
- Centered wheel
- Off-centered wheel
- Omni wheel

Wheels

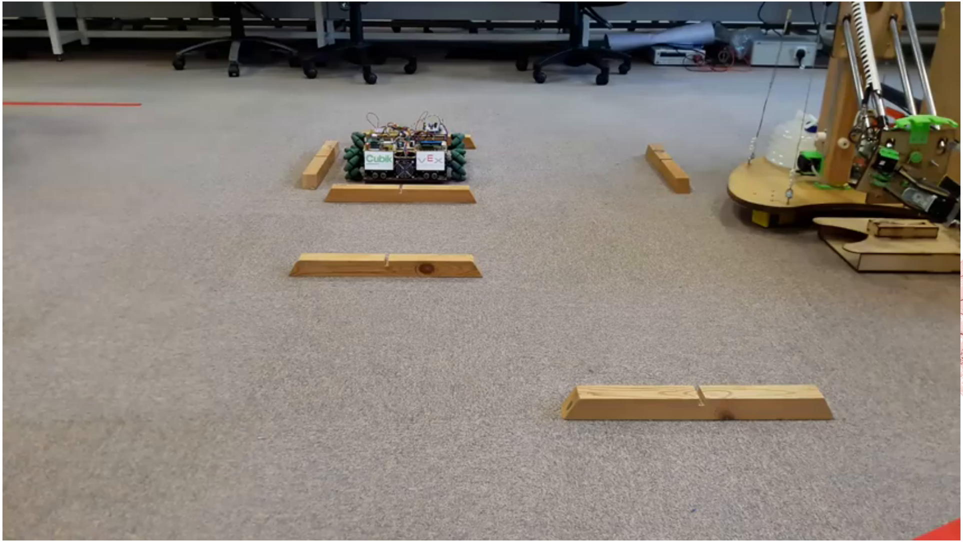
Types of wheels

- Fixed wheel
- Centered wheel
- Off-centered wheel
- Omni wheel
- Mecanum wheel



Wheels

Mecanum wheels



Wheeled robots

Wheel configuration

Wheeled robots are categorized based on the type of wheels and configurations that they use.

- By-wheel
- Tricycle
- Four wheel
- Omnidirectional
- etc. etc.

Kinematics

Wheeled robots

Comparison to other types

Kinematics

Description of robot pose in a inertial frame

Kinematics

Description of robot pose in a inertial frame

Pose

Wheeled robots

Comparison to other types

Kinematics

Description of robot pose in a inertial frame

Pose

Position

Kinematics

Description of robot pose in a inertial frame

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Orientation

Kinematics

Instantaneous center of rotation

Every motion can be modeled as a rotation around a point. For a circular motion, this point is fixed, but for a more complex motion it is constantly moving.

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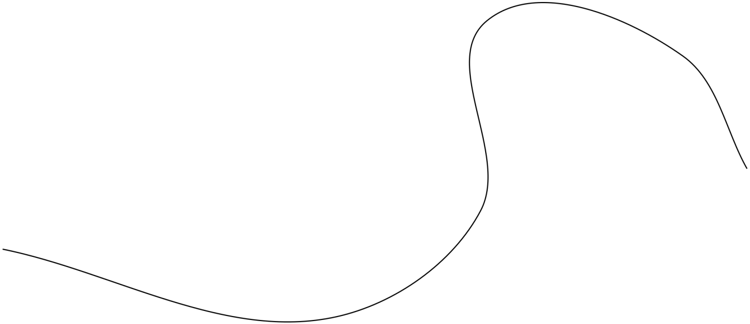

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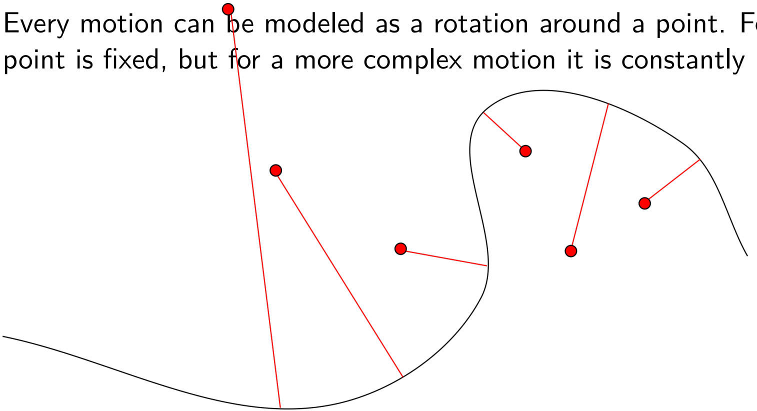

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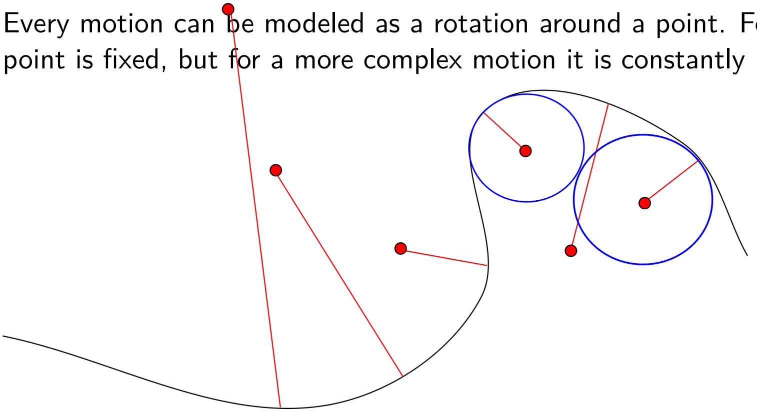

Kinematics

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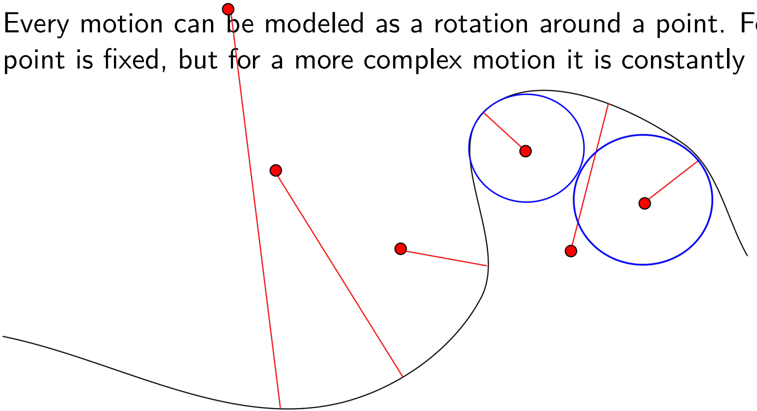


Where is the ICR for straight motion?

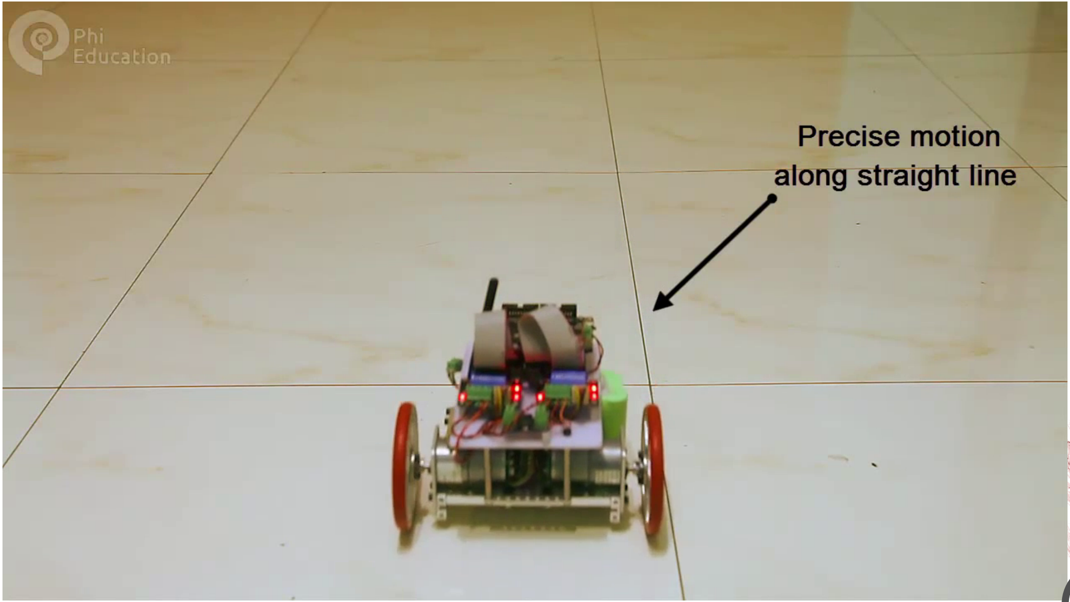
Wheeled robots

Differential drive

The differential drive is implemented:

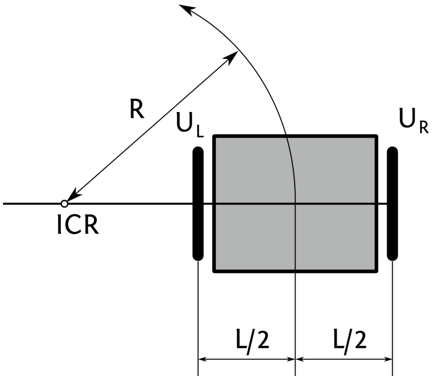
- Two driving wheels
- Each can rotate independently
- Need for a third balancing point (usually a roller-ball)
- Sensitive to relative velocity of the two wheels
- No sliding!

Differential drive



Differential drive

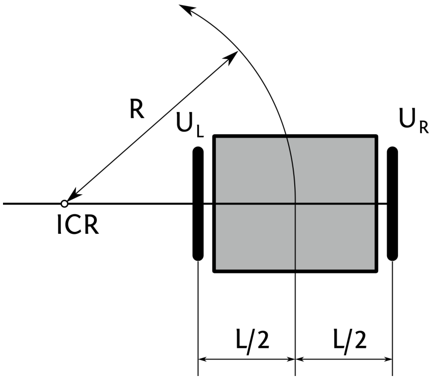
Kinematics modeling



Definitions:

Ω: Angular velocity of the robot
U: Linear velocity of the robot
U_i: Linear velocity at wheel i
ω_i: Angular velocity of wheel i
r: nominal radius of each wheel
R: Instantaneous Curvature Radius

Differential drive
Kinematics modeling



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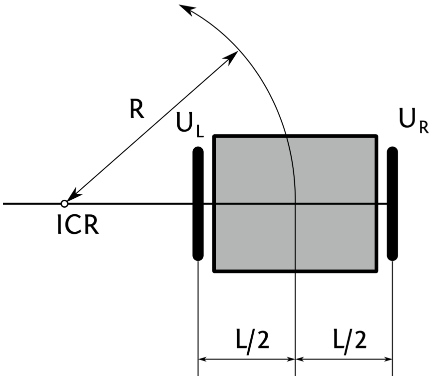
How many degrees of freedom?

Differential drive

Kinematics modeling



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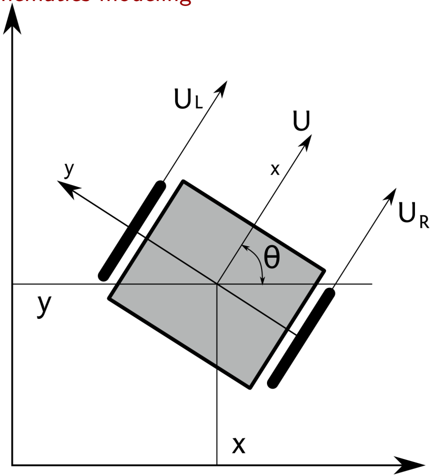
How many degrees of freedom? Mobile robots are non-holonomic!

Differential drive

Kinematics modeling



Pose of the robot

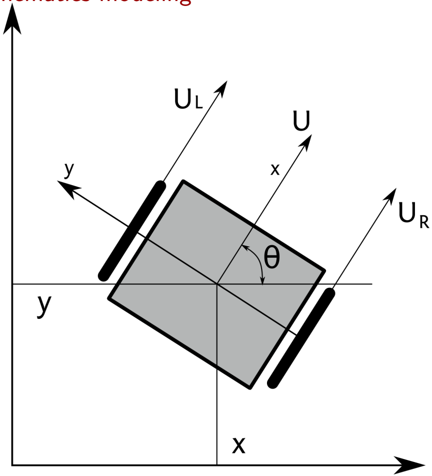
$$P = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$

Control input

$$U = \begin{bmatrix} U \\ \Omega \end{bmatrix}$$

Differential drive

Kinematics modeling



Pose of the robot

$$P = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$

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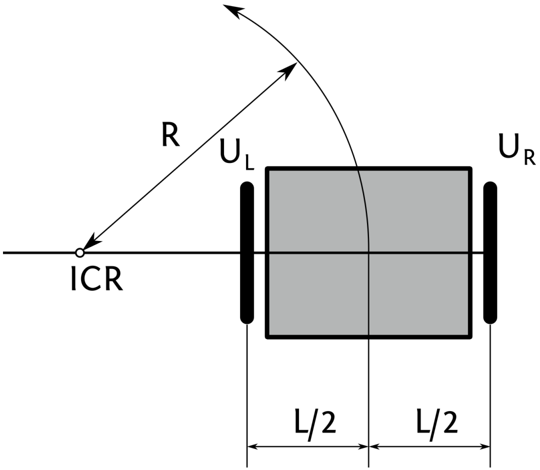
If we want to follow a specific trajectory (i.e. a specific R), the wheels must move in such rate so they rotate around the ICR with the same angular velocity

Differential drive

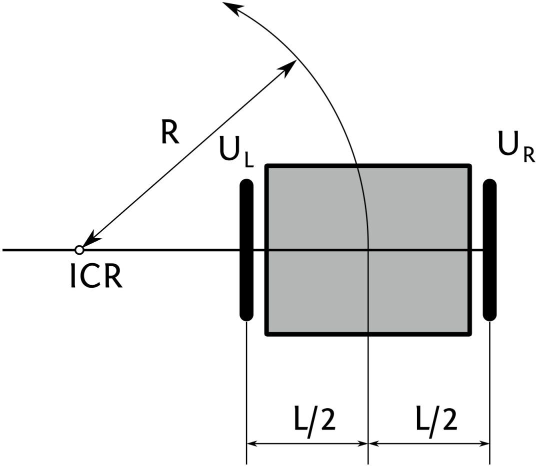
Forward kinematics modeling



$$\Omega = \frac{U_r}{R + \frac{L}{2}} = \frac{U_l}{R - \frac{L}{2}}$$

Differential drive

Forward kinematics modeling

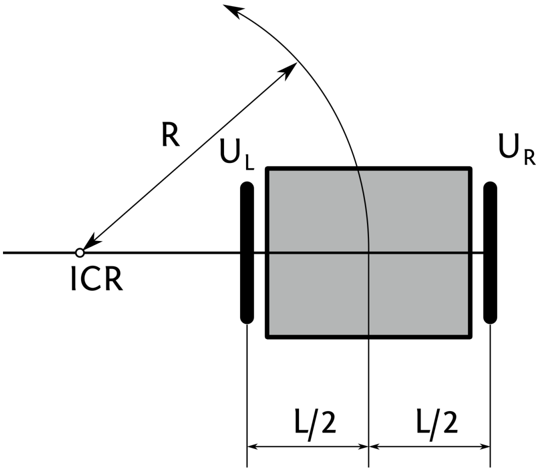


$$\Omega = \frac{U_r}{R + \frac{L}{2}} = \frac{U_l}{R - \frac{L}{2}}$$

$$R = \frac{L}{2}\frac{U_r + U_l}{U_r - U_l}$$

Differential drive

Forward kinematics modeling



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$$U = \Omega R$$

Differential drive

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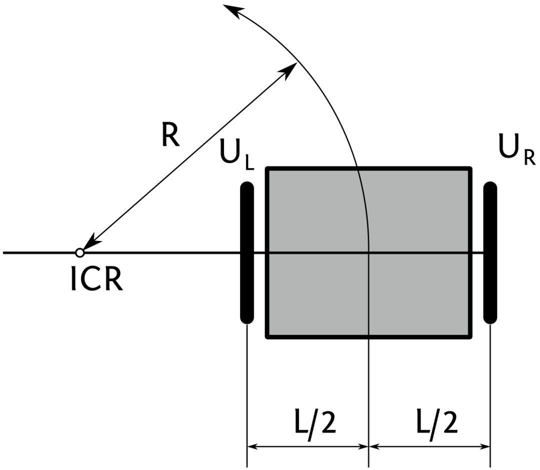

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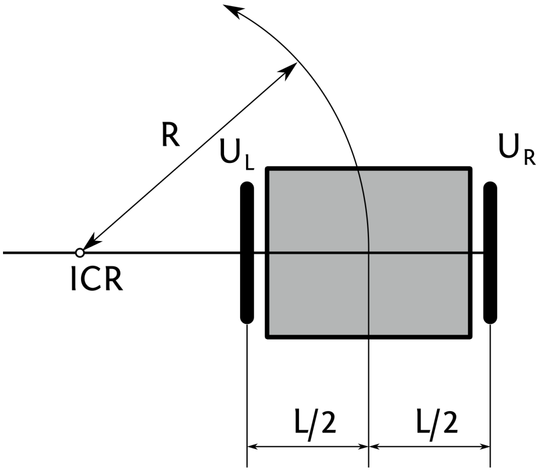
System of three equations with two unknowns

Differential drive

Forward kinematics modeling



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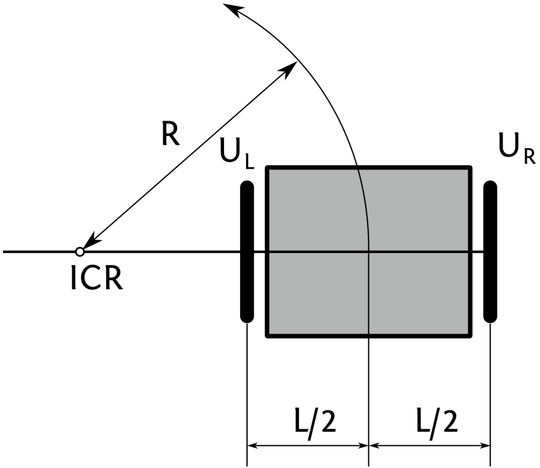
System of three equations with two unknowns

We determine either U_L and U_R, or any two from Ω, R, and U

Differential drive

Forward kinematics modeling

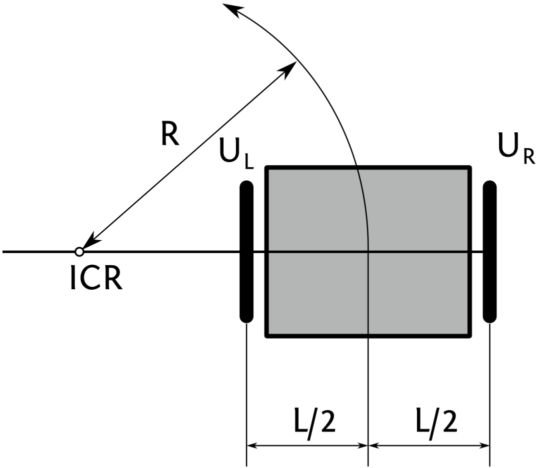


We can observe:

$$U = \frac{U_r + U_l}{2}$$

Differential drive

Forward kinematics modeling



We can observe:

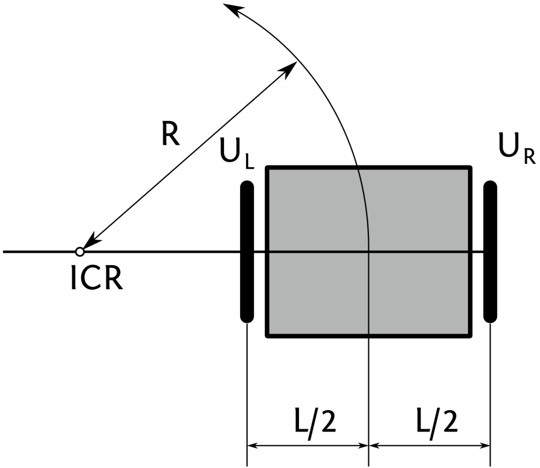
$$U = \frac{U_r + U_l}{2}$$

Knowing that:

$$R = \frac{L}{2}\frac{U_r + U_l}{U_r - U_l}$$

and:

$$u = \omega r$$



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and:

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$$\Omega = \frac{U_r - U_l}{L} = \frac{(\omega_r - \omega_l)r}{L}$$

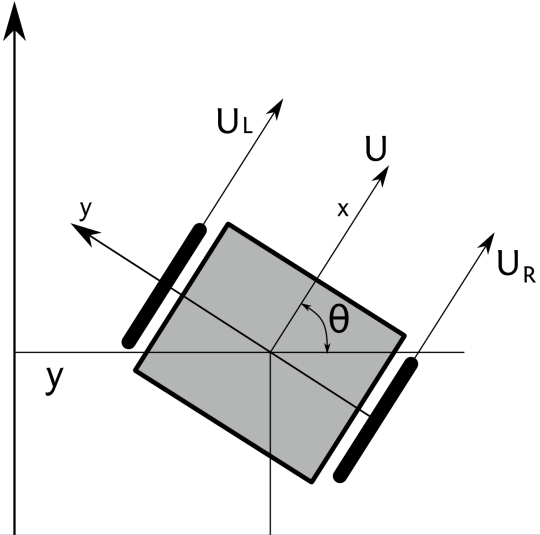
$$U = \frac{(\omega_r + \omega_l)r}{2}$$

What happens when $U_r = U_l$ OR $U_r = -U_l$?

Differential drive

Forward kinematics modeling

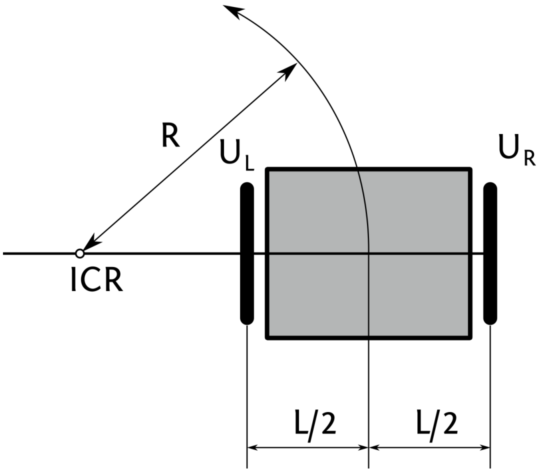
Kinematics model in the robot frame



$$\begin{bmatrix} U \\ \Omega \end{bmatrix} = \begin{bmatrix} \frac{r}{2} & \frac{r}{2} \\ \frac{r}{L} & \frac{-r}{L} \end{bmatrix} \begin{bmatrix} \omega_r \\ \omega_l \end{bmatrix}$$

Differential drive

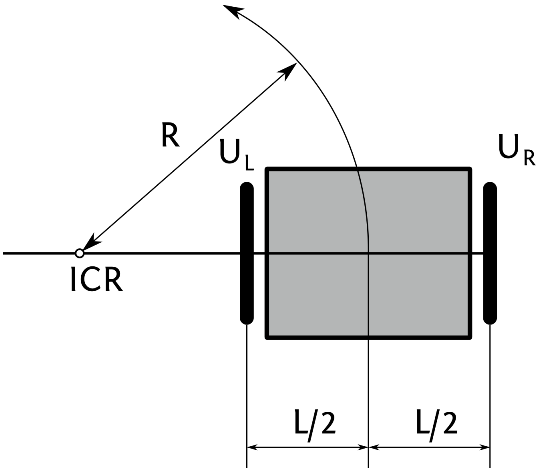
Inverse kinematics modeling



How do we define the inverse kinematics?

Differential drive

Inverse kinematics modeling

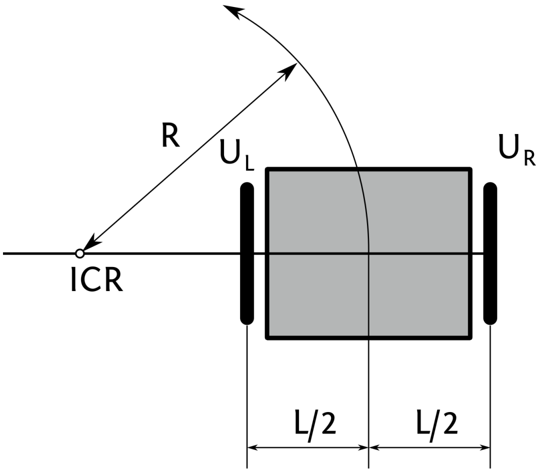


How do we define the inverse kinematics?

$$U_r = \Omega(R + \frac{L}{2}) = U(1 + \frac{L}{2R})$$
$$U_l = \Omega(R - \frac{L}{2}) = U(1 - \frac{L}{2R})$$

Differential drive

Inverse kinematics modeling



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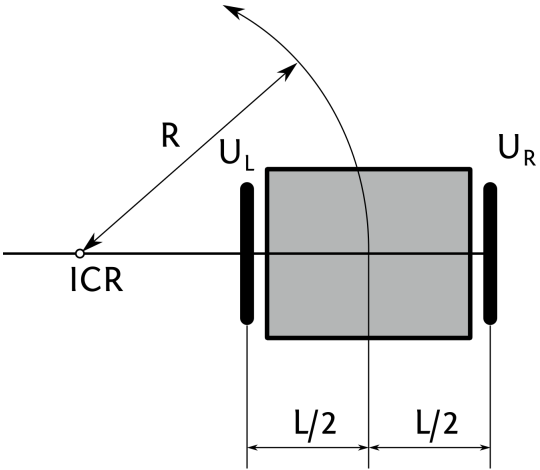
Where:

$u_r = r\omega_r$

$u_l = r\omega_l$

Differential drive

Inverse kinematics modeling



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Where:

$$u_r = r\omega_r$$
$$u_l = r\omega_l$$

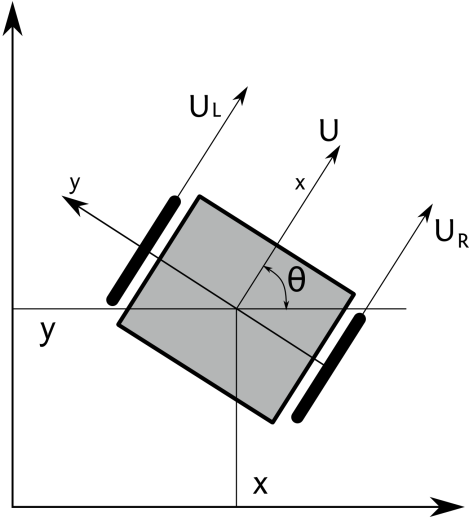
Therefore:

$$\omega_r = \Omega\frac{R + \frac{L}{2}}{r} = U\frac{1 + \frac{L}{2R}}{r}$$
$$\omega_l = \Omega\frac{R - \frac{L}{2}}{r} = U\frac{1 - \frac{L}{2R}}{r}$$

Differential drive

Kinematics modeling

Kinematics model in the world frame



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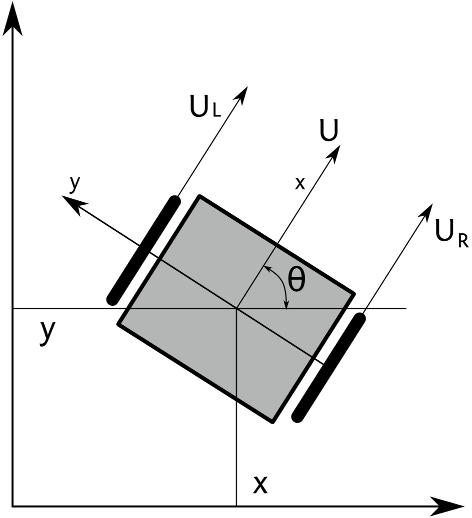
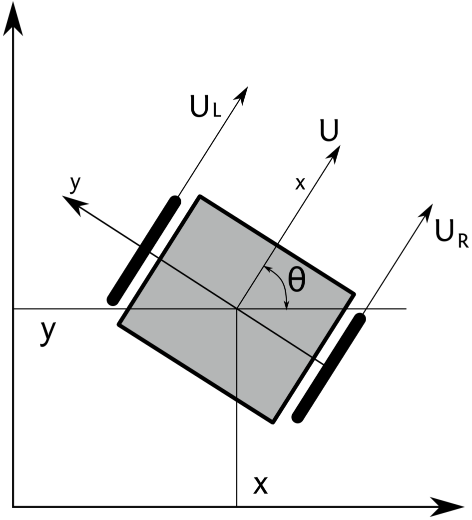


$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\theta}\end{bmatrix} = \begin{bmatrix} cos\theta & 0\\ sin\theta & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}U\\ \Omega\end{bmatrix}$$

Differential drive

Kinematics modeling

Kinematics model in the world frame



$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} cos\theta & 0 \\ sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} U \\ \Omega \end{bmatrix}$$

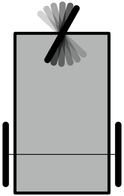
How do we calculate velocity in world frame with respect to ω_i?

Tricycle

Description

A wheeled robot with three wheels:

- Two fixed wheels with the same axis
- The two wheels can move independently
- One wheel that steers and pushes the robot
- The third wheel is usually between the other two with an offset

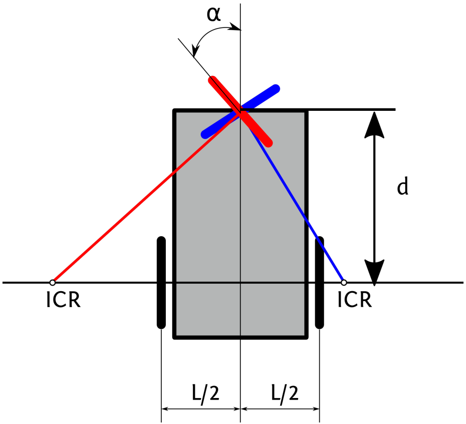


Tricycle

Tricycle

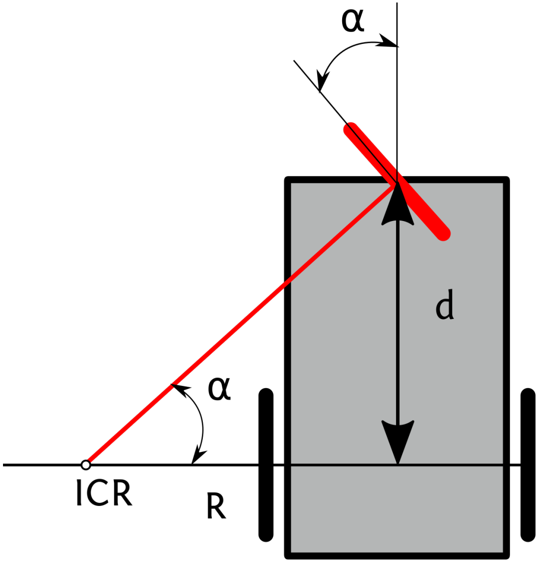
Kinematics

We control the location of the ICR by changing the steering angle α, and the velocity, by changing the wheel velocity ω_w



Tricycle

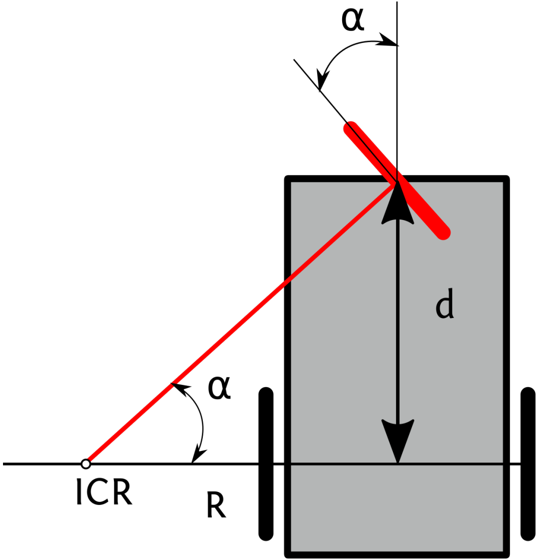
Forward kinematics model



If r is the steering wheel radius, then:

Tricycle

Forward kinematics model



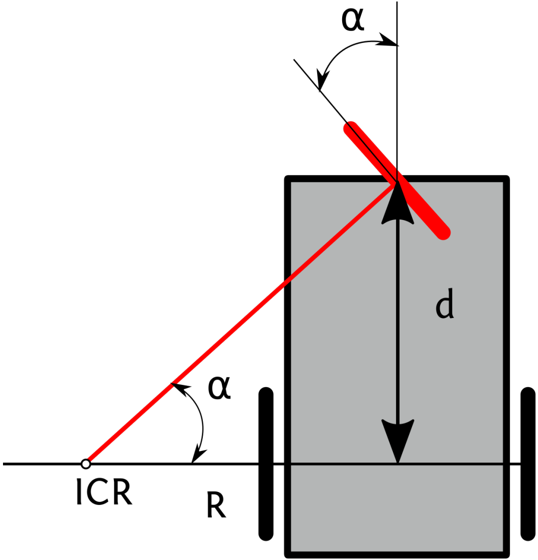
If r is the steering wheel radius, then:

$u_w = \omega_w r$

$R = d * tan(\frac{\pi}{2} - \alpha)$

Tricycle

Forward kinematics model



If r is the steering wheel radius, then:

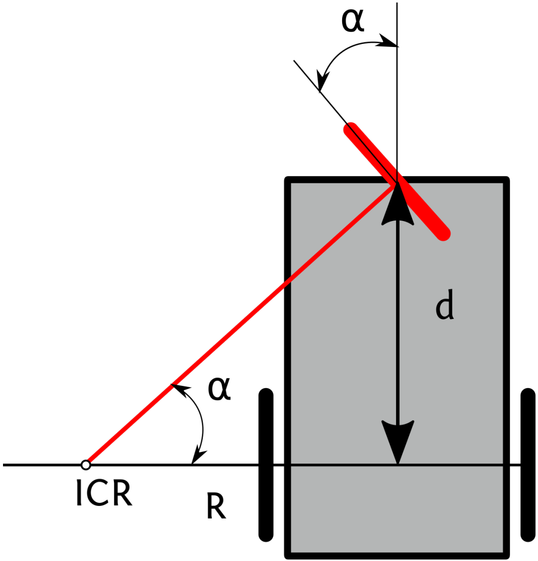
$$u_w = \omega_w r$$
$$R = d * tan(\frac{\pi}{2} - \alpha)$$

The angular velocity of the robot relative to the base frame:

$$\Omega = \frac{u_w}{\sqrt{d^2 + R^2}}$$

Tricycle

Forward kinematics model



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The angular velocity of the robot relative to the base frame:

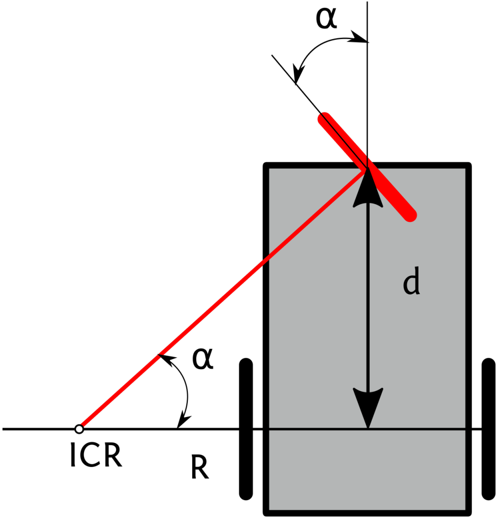
$$\Omega = \frac{u_w}{\sqrt{d^2 + R^2}}$$

$$\Omega = \frac{u_w}{d} sin(\alpha)$$

Tricycle

Forward kinematics model

Kinematics model in the robot body frame:



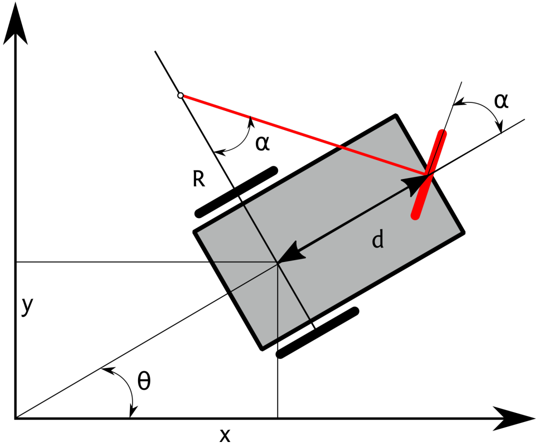
$$U = u_w cos(\alpha)$$

$$\Omega = \frac{u_w}{d} sin(\alpha)$$

Tricycle

Forward kinematics model

Kinematics model in the world body frame:



$$\dot{x} = u_w cos(\alpha)\cos(\theta)$$

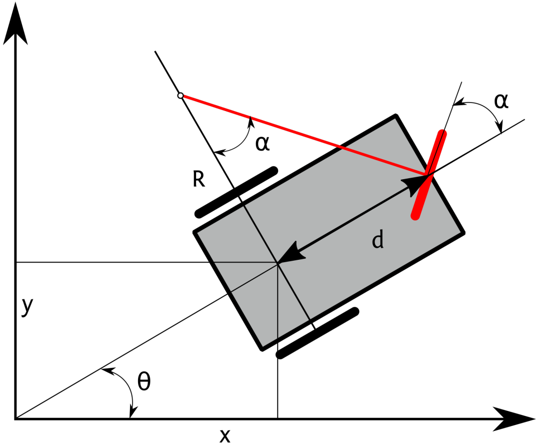
$$\dot{y} = u_w cos(\alpha)\sin(\theta)$$

$$\dot{\theta} = \omega = \frac{u_w}{d} sin(\alpha)$$

Tricycle

Forward kinematics model

Kinematics model in the world body frame:



$$\dot{x} = u_w cos(\alpha)\cos(\theta)$$

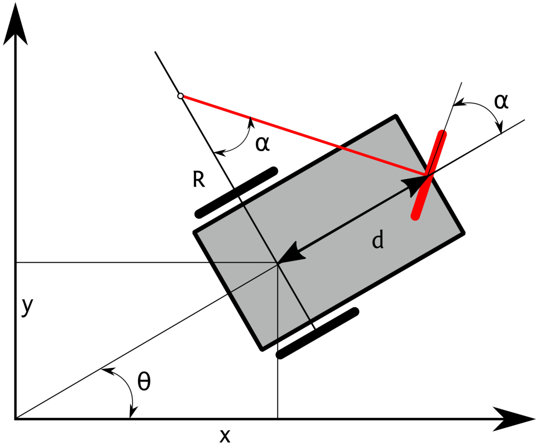
$$\dot{y} = u_w cos(\alpha)\sin(\theta)$$

$$\dot{\theta} = \omega = \frac{u_w}{d} sin(\alpha)$$

What about the inverse?

Tricycle

Inverse kinematics model



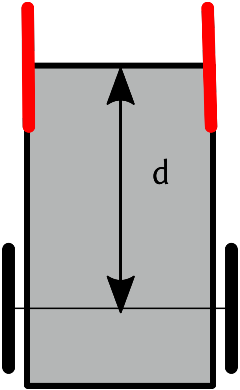
$$\alpha = atan(\frac{\dot{\theta} d sin\theta}{\dot{y}})$$

$$u_w = \frac{\dot{y}}{cos(\alpha)sin(\theta)}$$

Four wheels

Description

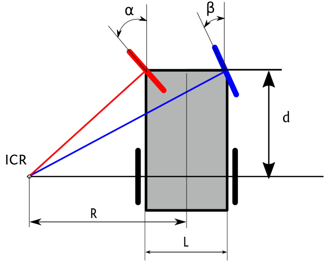
Another type of wheeled robot, is with four wheels. The two front are transmitting the power and are steered, while the back ones are fixed wheels



Four wheels

Ackerman drive

For this to work, the steering of the two wheels must be coordinated:



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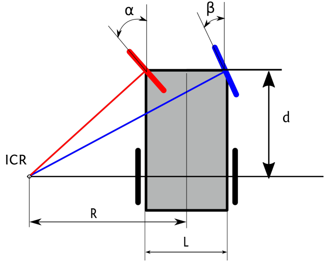
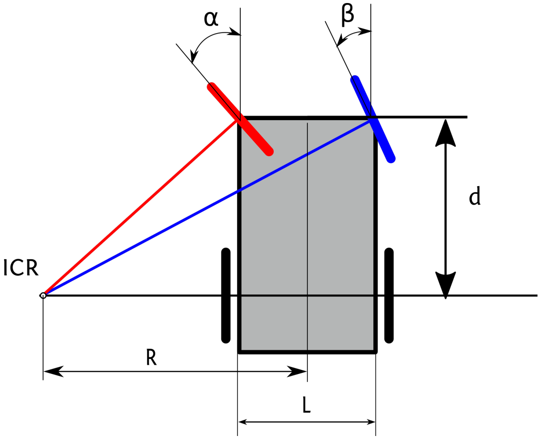


$\alpha > \beta$: when turning left

$\beta > \alpha$: when turning right

Four wheels

Ackerman drive



Four wheels

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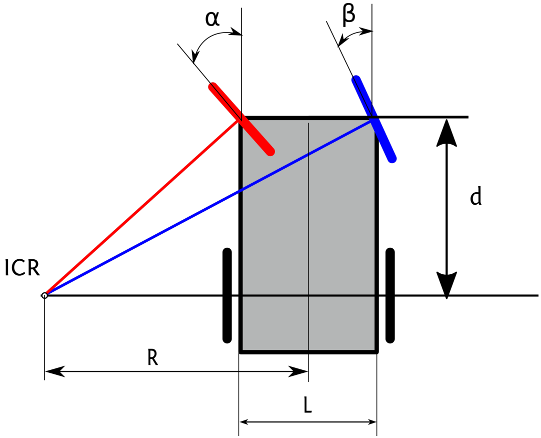
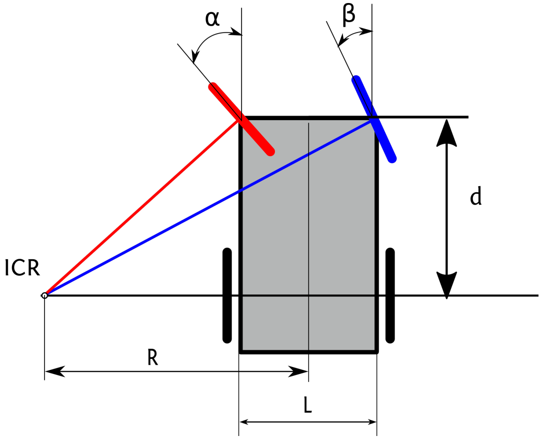


$$cot(\alpha) = \frac{R - \frac{L}{2}}{d}$$

$$cot(\beta) = \frac{R + \frac{L}{2}}{d}$$

Four wheels

Ackerman drive



$$cot(\alpha) = \frac{R - \frac{L}{2}}{d}$$

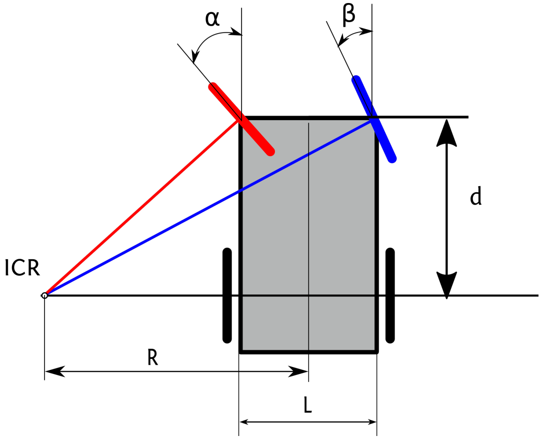
$$cot(\beta) = \frac{R + \frac{L}{2}}{d}$$

Therefore:

$$cot(\beta) - cot(\alpha) = \frac{L}{d}$$

Four wheels

Ackerman drive



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What happens when $\alpha = \beta = 0$?

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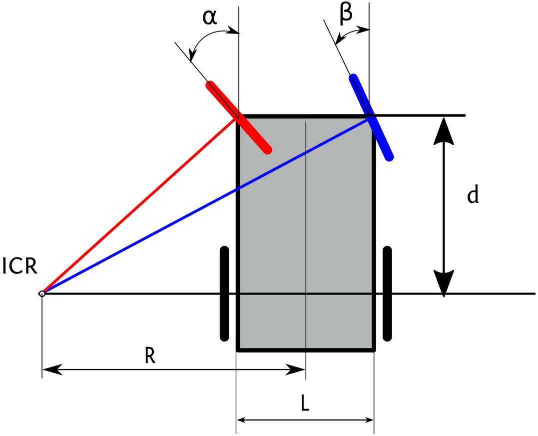

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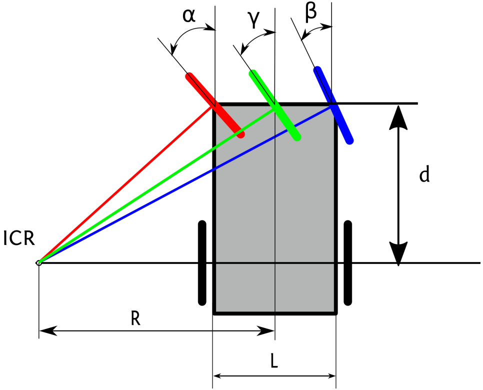
What happens when $\alpha = \beta = 0$?

What is the relationship between angular velocities ω_l and ω_r?

Four wheels

Ackerman and tricycle

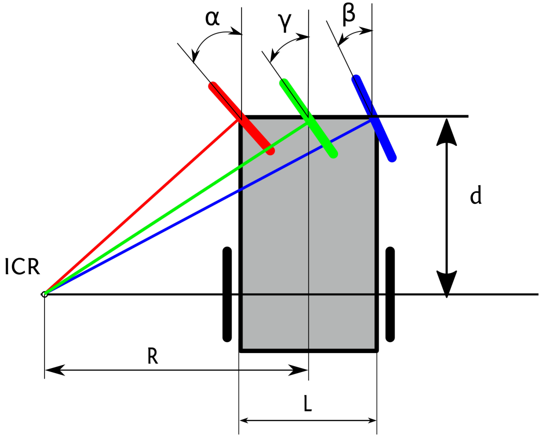
We can describe the ackerman drive kinematics, the same way as for the tricycle, if we consider a virtual fifth wheel between the two front ones



Four wheels

Ackerman drive

We can easily calculate the equivalent virtual angle γ



Four wheels

Ackerman drive

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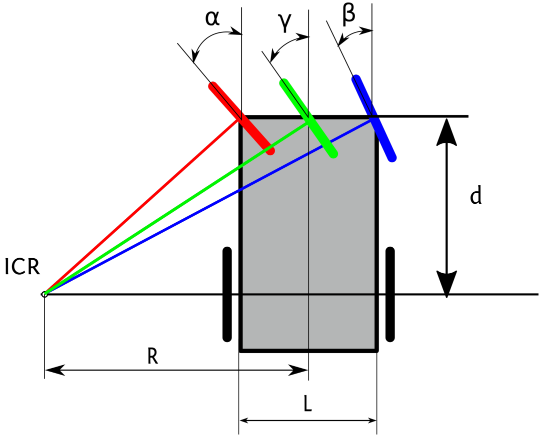

$$cot(\gamma) = cot(\alpha) + \frac{L}{2d}$$

$$cot(\gamma) = cot(\beta) - \frac{L}{2d}$$

Four wheels

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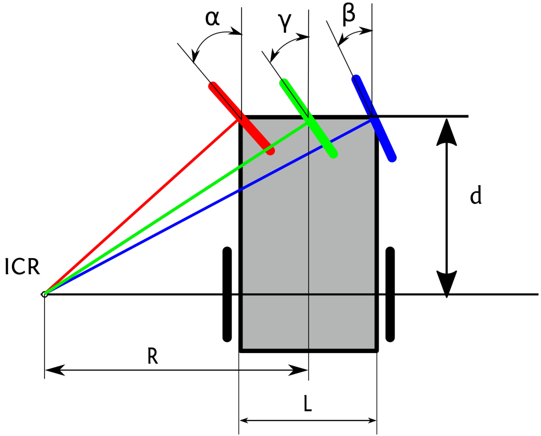

$$cot(\gamma) = cot(\alpha) + \frac{L}{2d}$$

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The kinematics models then are the same as for a tricycle with steering angle γ

Four wheels

Skid steer drive

The skid steer drive consists of four individually driven wheels, all with a fixed direction

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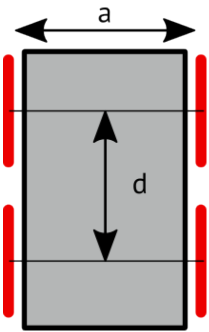

Four wheels

Skid steer drive

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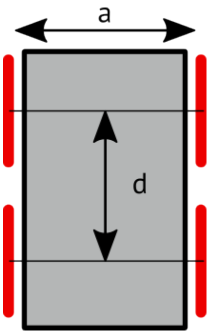

What is the issue with this design?

Four wheels

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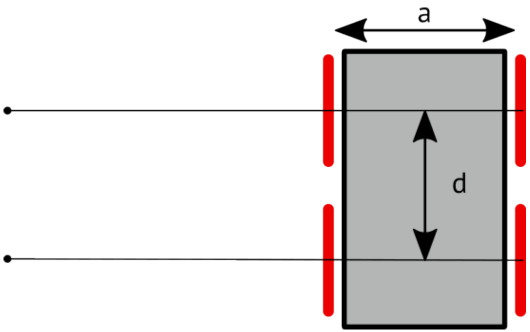

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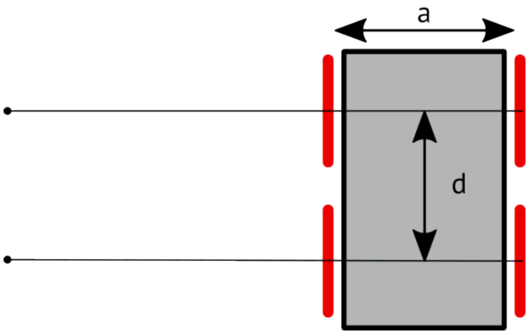

Two different ICR for the robot!

Skid steer drive

Issues

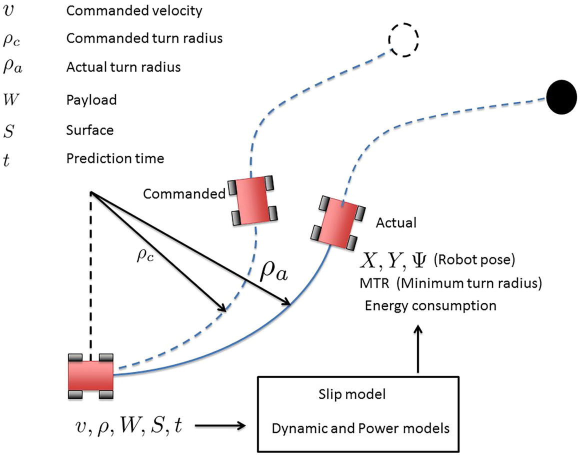

Figure: From: Learning of skid-steered kinematic and dynamic models for motion planning, Ordonez et. al.

Skid steer drive

Modelling

We assume a differential drive model:

$$\begin{bmatrix} U \\ \Omega \end{bmatrix} = \begin{bmatrix} \frac{r}{2} & \frac{r}{2} \\ \frac{r}{L} & \frac{-r}{L} \end{bmatrix} \begin{bmatrix} \omega_r \\ \omega_l \end{bmatrix}$$

Skid steer drive

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We add one more dimension for lateral movement (slip, u_l)

Skid steer drive

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$$\begin{bmatrix} U_f \\ U_l \\ \Omega \end{bmatrix} = \begin{bmatrix} \frac{r}{2} & \frac{r}{2} \\ 0 & 0 \\ \frac{r}{L} & \frac{-r}{L} \end{bmatrix} \begin{bmatrix} \omega_r \\ \omega_l \end{bmatrix}$$

Skid steer drive

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We add one more dimension for lateral movement (slip, u_l)
And we consider some disturbances on each direction

$$\begin{bmatrix} U_f \\ U_l \\ \Omega \end{bmatrix} = \begin{bmatrix} \frac{r}{2} & \frac{r}{2} \\ 0 & 0 \\ \frac{r}{L} & \frac{-r}{L} \end{bmatrix} \begin{bmatrix} \omega_r \\ \omega_l \end{bmatrix} + \begin{bmatrix} \delta u_f \\ \delta u_l \\ \delta \omega \end{bmatrix}$$

Skid steer drive

Modelling slipage

There are different ways to model these *disturbances*:

Skid steer drive

Modelling slipage

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$$
\begin{aligned}
\delta u_f &= \alpha_{11} u_c + \alpha_{12} \omega_c + \alpha_{13} u_c \omega_c \\
\delta u_l &= \alpha_{21} u_c + \alpha_{22} \omega_c + \alpha_{23} u_c \omega_c \\
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Skid steer drive

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or:

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\begin{bmatrix} \delta u_f \\ \delta u_l \\ \delta \omega \end{bmatrix} = A \begin{bmatrix} u_c \\ \omega_c \\ u_c \omega_c \end{bmatrix}
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Skid steer drive

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What does the matrix A depend on?

Skid steer drive

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What does the matrix A depend on?
How do we calculate it?

Skid steer drive

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$$

What does the matrix A depend on?
How do we calculate it?
What do we do when we cannot rely on it?

Inertial Measurement Unit

IMU

When we cannot rely on characterising matrix A, we need external sensors for feedback.

Inertial Measurement Unit

IMU

When we cannot rely on characterising matrix A, we need external sensors for feedback.

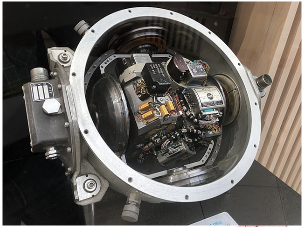


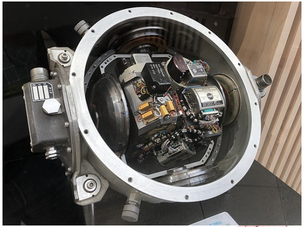
Figure: Inertial Measurement Unit of the apollo missions

Inertial Measurement Unit

IMU

When we cannot rely on characterising matrix A, we need external sensors for feedback.

IMUs provide information on acceleration, rotational rate, and sometimes magnetic direction



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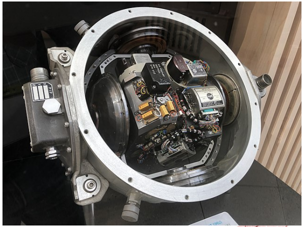
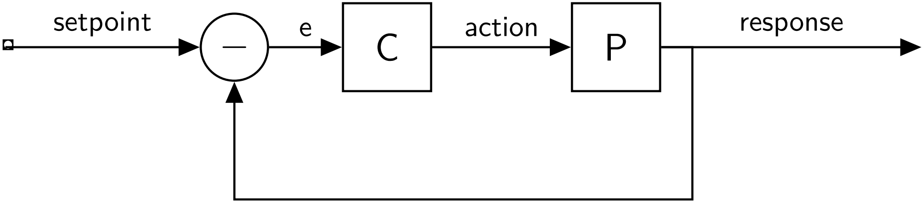
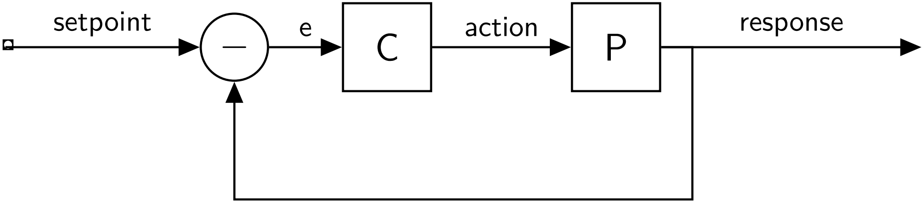


Figure: Inertial Measurement Unit of the apollo missions

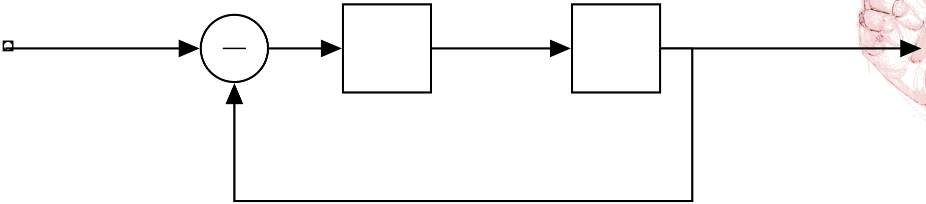
Basic control



Basic control

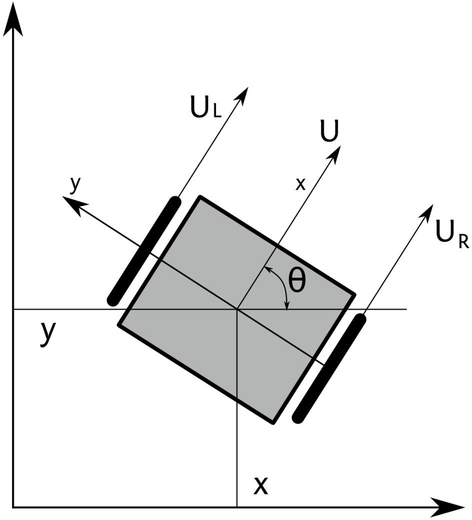


Let's fill in the blanks



Dynamic modelling

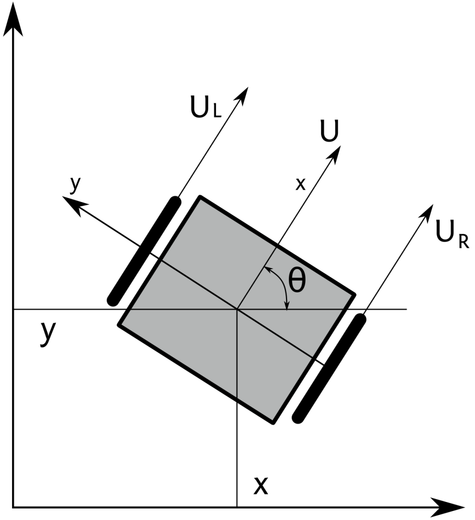
Differential drive



Generalised coordinates?

Dynamic modelling

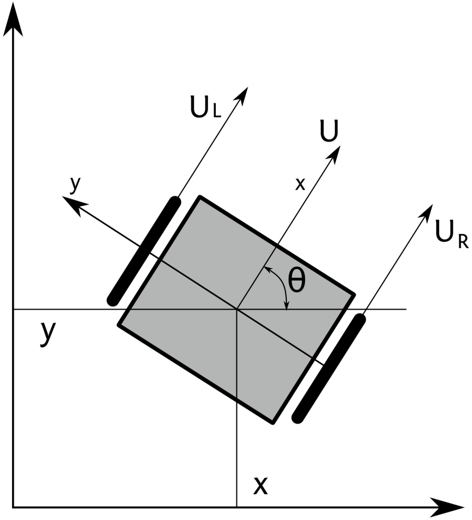
Differential drive



Generalised coordinates?
Euler-Lagrange?

Dynamic modelling

Differential drive

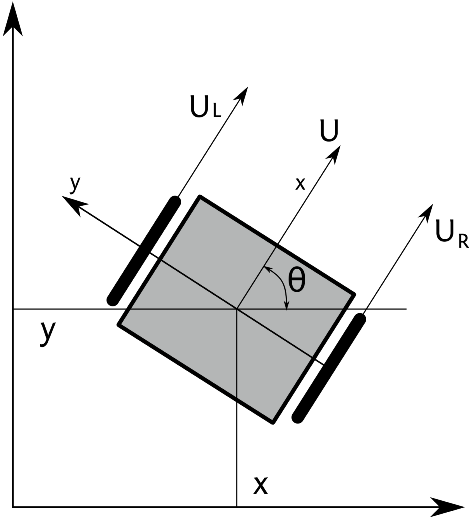


Generalised coordinates?
Euler-Lagrange?

$$q = \begin{bmatrix} X & Y & \theta & \phi_l & \phi_r \end{bmatrix}$$

Dynamic modelling

Differential drive

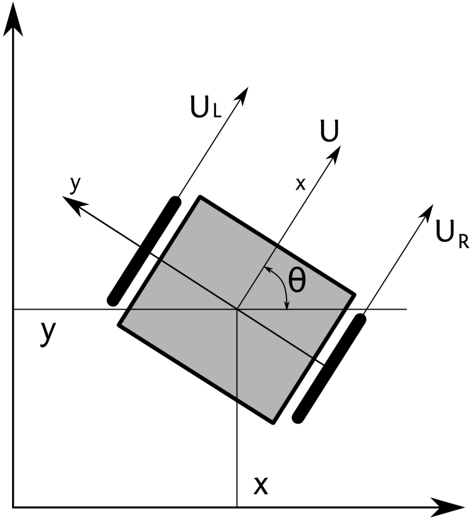


$$L = T - V$$

$$T = T_{lin} + T_{ang}$$

Dynamic modelling

Differential drive



$$L = T - V$$

$$T = T_{lin} + T_{ang}$$

$$T = T_{robot} + T_{wl} + T + wr$$

Questions?